Describing Rotational Motion

Pre-Lecture Questions

Problem Set #25 (due next time)

Lecture Outline

1. The Definitions of the Rotational Variables
2. Rotational Kinematics

Pre-Class Summary:

Translational Quantity	Rotational Quantity
Position: The location of an object with respect to a coordinate system.	Angle: The rotational location of an object with respect to a coordinate system.
Displacement: A change in position.	Angular Displacement: A change in angle.
Velocity: The rate of displacement.	Angular Velocity: The rate of angular displacement.
Acceleration: The rate of change of velocity.	Angular Acceleration: The rate of change of angular velocity.

Pre-Class Summary:

Translational Variables	Rotational Variables	Relationship
Position: x	Angle: θ	$s = r\theta$
Displacement: dx	Angular Displacement: dθ	$ds = r d\theta$
Velocity: $v \equiv \frac{dx}{dt}$	Angular Velocity: $\omega \equiv \frac{d\theta}{dt}$	$v_t = r\omega$
Acceleration: $a \equiv \frac{dv}{dt}$	Angular Acceleration: $\alpha \equiv \frac{d\omega}{dt}$	$a_t = r\alpha$ $a_c = \omega^2 r$

Pre-Class Summary:

$$\left.\begin{array}{l} v \equiv \dfrac{dx}{dt} \\ a \equiv \dfrac{dv}{dt} \\ \dfrac{da}{dt} = 0 \end{array}\right\} \text{a bunch of math} \Rightarrow \left\{\begin{array}{c} v = v_o + at \\ x = x_o + v_o t + \tfrac{1}{2} a t^2 \\ v^2 = v_o^2 + 2a(x - x_o) \\ x - x_o = \tfrac{1}{2}(v + v_o)t \end{array}\right.$$

$$\left.\begin{array}{l} \omega \equiv \dfrac{d\theta}{dt} \\ \alpha \equiv \dfrac{d\omega}{dt} \\ \dfrac{d\alpha}{dt} = 0 \end{array}\right\} \text{the very same math} \Rightarrow \left\{\begin{array}{c} \omega = \omega_o + \alpha t \\ \theta = \theta_o + \omega_o t + \tfrac{1}{2} \alpha t^2 \\ \omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o) \\ \theta - \theta_o = \tfrac{1}{2}(\omega + \omega_o)t \end{array}\right.$$

The units for angular quantities are often confusing. Let's work on some units associated with angles.

1. At the right, draw a bicycle wheel including the spokes.

2. Imagine one spoke rotating around. It makes an angle Ø with its original position. Draw this for some angle Ø less than 90°.

3. Now imagine the spoke completing exactly one rotation. Find the angle in revolutions, degrees, and radians.

4. Write the conversion factors for revolutions to degrees, degrees to radians, and radians to revolutions.

   1 rev = ________________°

   __________° = _______ rads

   _______ rads = _______ rev

5. Convert 30° to revolutions and radians.

   30° = ________________ rev

   30° = ________________ rad

Now, let's think about the rate things spins.

1. At the right, draw a bicycle wheel including the spokes. Indicate that it is spinning.

2. Imagine one spoke rotating around. It completes one rotation in a time T which could be in minutes or seconds. Write it's angular speed in degrees per time, revolutions per time and radians per time.

ω = ________(°/m)

ω = ________(rev/m)

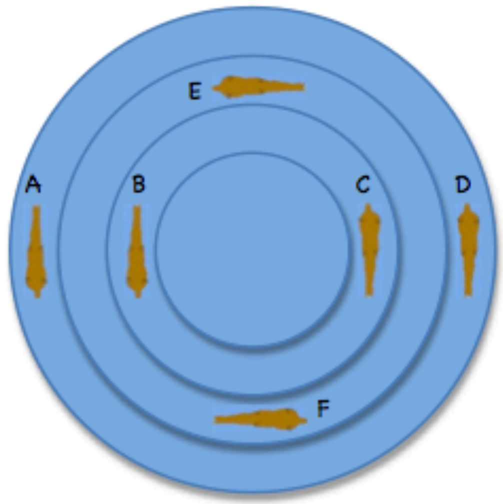
ω = ________(rad/s)

3. Convert one revolution per minute to radians per second.

1rpm = ____________ rad/s

Example 1: A batter swings a 90.0cm long bat such that it is in circular motion about the end. The center-of-mass of the bat is 60.0cm from the end and starts from rest and reaches a speed of 30.0m/s in 65.0ms. Find (a)the final angular speed in rad/s and rpm, (b)the average angular acceleration, and (c)the acceleration of the center-of-mass after the 65.0ms.

At the right is a sketch of the top view of a merry go round that spins at a constant rate.



Rank these horses from greatest to least based upon their:

1. angular speed.

2. tangential speed.

3. angular acceleration.

4. tangential acceleration.

5. centripetal acceleration.

Example 2:A batter swings a 90.0cm long bat such that it is in circular motion about the end. The center-of-mass of the bat is 60.0cm from the end and starts from rest and reaches a speed of 30.0m/s in 65.0ms. Find (a)the angular displacement in radians and revolutions and (b)the distance traveled by the end of the bat.

Lecture 25 - Summary

The Definitions of Rotational Variables

Rotational Quantity	Mathematically
Angle: The rotational location of an object with respect to a coordinate system.	θ
Angular Displacement: A change in angle.	$d\theta$
Angular Velocity: The rate of angular displacement.	$\omega \equiv \frac{d\theta}{dt}$
Angular Acceleration: The rate of change of angular velocity.	$\alpha \equiv \frac{d\omega}{dt}$

Angular/Linear Rules

$$s = r\theta$$

$$v_t = r\omega$$

$$a_t = r\alpha$$

$$a_c = \omega^2 r$$

If the angular acceleration is constant the kinematic equations apply.

$$\omega = \omega_o + \alpha t$$

$$\theta = \theta_o + \omega_o t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$\theta - \theta_o = \tfrac{1}{2}(\omega + \omega_o)t$$