Torque as a Vector

Pre-Lecture Questions

Problem Set #28 (due next time)

Lecture Outline

1. Review of the Cross Product
2. The Torque Vector

Pre-Class Summary:

The vector cross product defined as,

$$\vec{a} \times \vec{b} \equiv ab\sin\theta\hat{n}$$

The cross product can be calculated mathematically using,

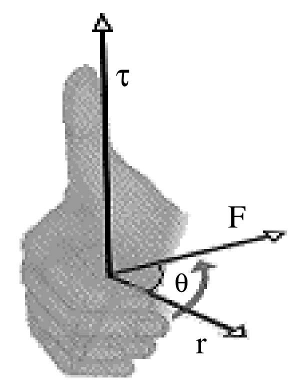
$$\vec{a} \times \vec{b} = \left(a_y b_z - a_z b_y\right)\hat{i} + \left(a_z b_x - a_x b_z\right)\hat{j} + \left(a_x b_y - a_y b_x\right)\hat{k}$$

or

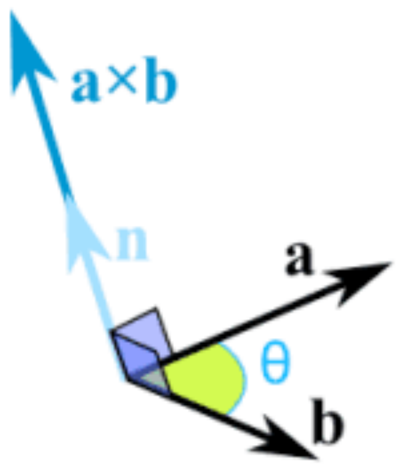
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}.$$

Since torque is a vector, we redefined it as,

$$\vec{\tau} \equiv \vec{r} \times \vec{F}$$



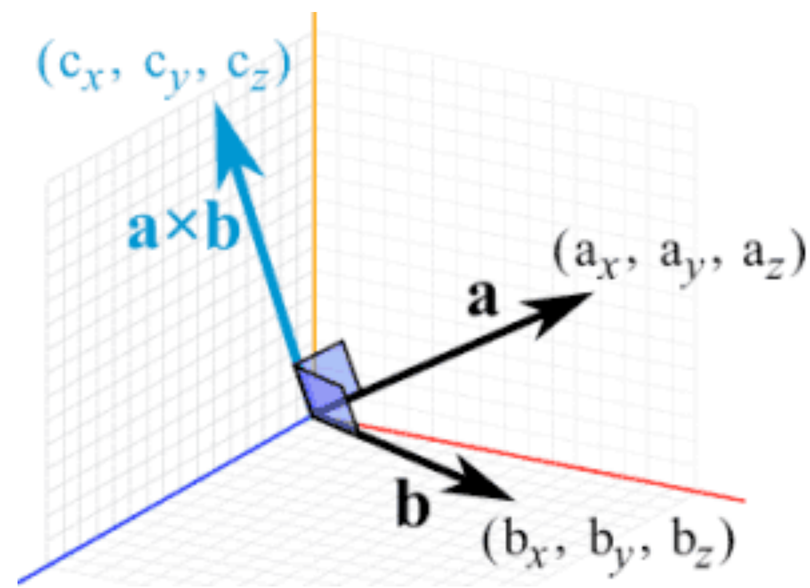
Review of the Cross Product



$$\vec{a}\times\vec{b} \equiv ab\sin\theta\hat{n}$$

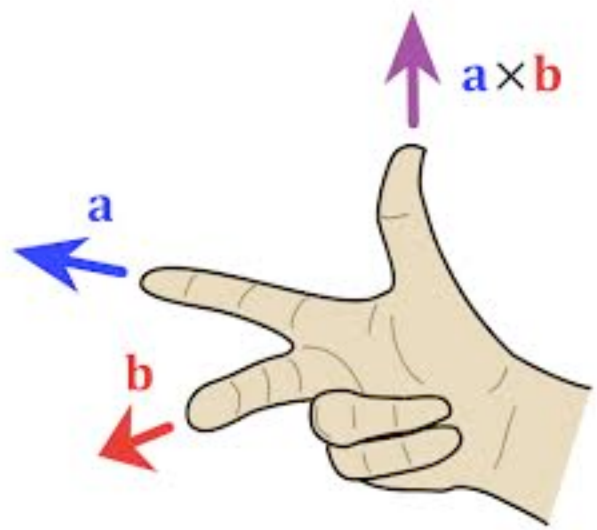
$$\vec{a}\times\vec{b} = \left(a_y b_z - a_z b_y\right)\hat{i} + \left(a_z b_x - a_x b_z\right)\hat{j} + \left(a_x b_y - a_y b_x\right)\hat{k}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$



The Right Hand Rule:

- point your fingers along the first vector.
- rotate your hand so that you can bend your fingers the shorter way around toward the second vector.
- your thumb points along the cross product.

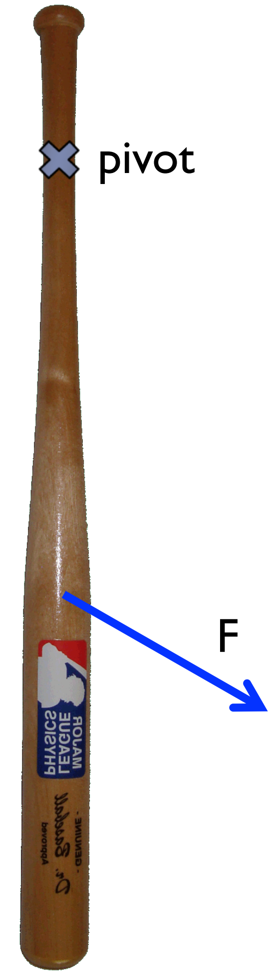


Example 1: Given the two vectors below, find (a)A×B, (b)B×A, (c)the angle between them, and (c)the component of B perpendicular to A.

$\vec{A} = 12\hat{i} + 5.0\hat{j}$ $\vec{B} = 6.0\hat{i} + 8.0\hat{j}$

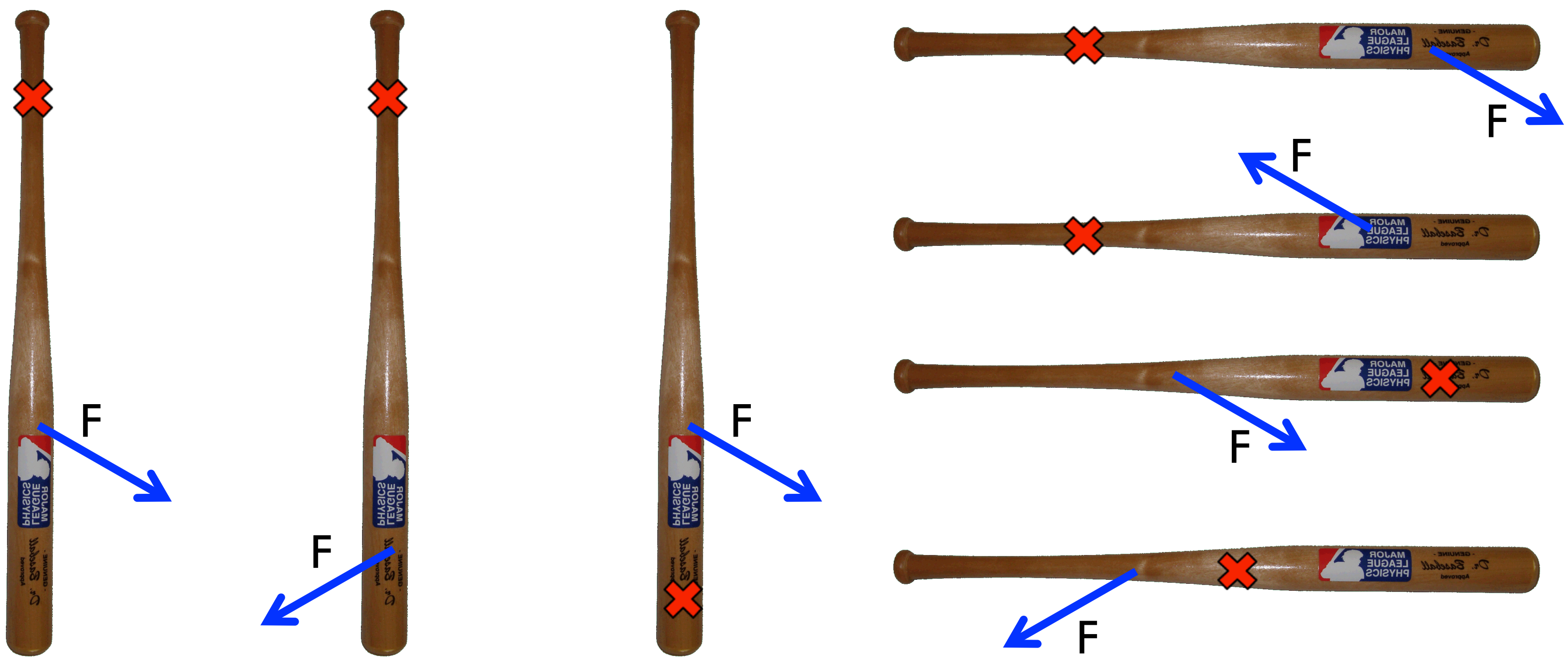
Torque and Cross Product

1. Draw the r vector.
2. Move the r vector so it's tail is at the tail of the F vector.
3. Label the angle between r and F as θ.
4. Draw the component of F that creates the torque.
5. Express this component in terms of F and θ.
6. Write the magnitude of the torque in terms of the magnitudes of r and F as well as θ.
7. Compare your answer with the magnitude of the cross product between the vectors r and F.
8. Explain in words what a cross product calculates and why it is perfect for defining torque.



A New Twist on Torque

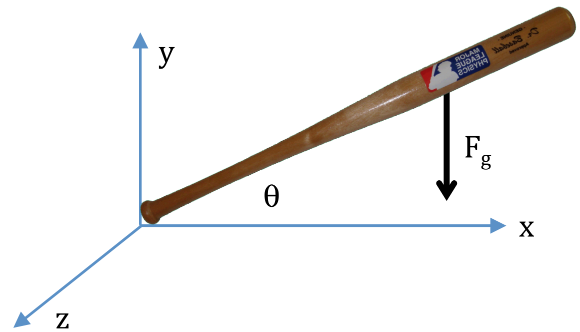
For each situation below, the pivot is indicated with a red x. Indicate the direction of the torque vector using the notation described at the bottom.



⊗ = into the page

⊙ = out of the page

Example 2: A 1.0kg - 90cm long bat has a center-of-mass 60cm from the handle end. It is held on the handle so that it points upward at a 30° angle. Find the torque exerted by gravity about (a)the handle end, (b)the barrel end, and (c)the center-of-mass.



Lecture 28 - Summary

We reviewed the mathematics of the vector cross product defined as,

$$\vec{a} \times \vec{b} \equiv ab \sin\theta \hat{n}$$

The cross product can be calculated mathematically using,

$$\vec{a} \times \vec{b} = \left(a_y b_z - a_z b_y\right)\hat{i} + \left(a_z b_x - a_x b_z\right)\hat{j} + \left(a_x b_y - a_y b_x\right)\hat{k}$$

or

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}.$$

The mathematics of the cross product led to a deeper understanding of the vector nature of torque redefined as,

$$\vec{\tau} \equiv \vec{r} \times \vec{F}$$