Hooke's Rule and Simple Harmonic Motion(SHM)

Pre-Lecture Questions

Problem Set #34 (due next time)

Lecture Outline

1. A Mass at the End of a Spring
2. The SHM Equations of Motion

Pre-Class Summary:

The force exerted by a spring is given by Hooke's Rule $\vec{F}_s = -k\vec{x}$. Then the Second Law gives an acceleration of $a = -\frac{k}{m}x$ resulting in the

Equations of Motion for SHM:

$$a(x) = -\omega^2 x \qquad x(t) = A\cos(\omega t + \delta)$$

$$v(t) = -\omega A\sin(\omega t + \delta)$$

$$v(x) = \pm\omega\sqrt{A^2 - x^2} \qquad a(t) = -\omega^2 A\cos(\omega t + \delta)$$

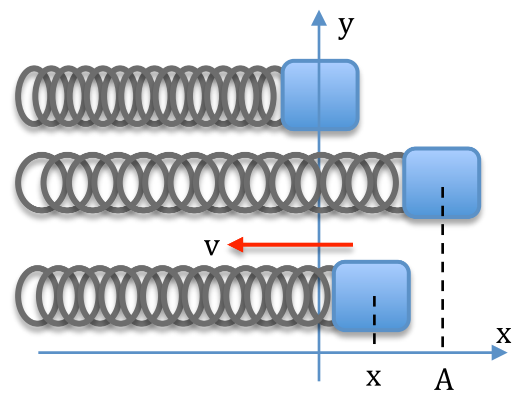
where A is the amplitude of the motion, ω is the angular frequency, and δ is the phase angle.

All of these equations follow from the first one. So any time we have a system where the acceleration is equal to the negative of a constant times the position, we will get this SHM and the angular frequency will be equal to the root of the constant.

For a mass on the end of a spring, $\omega = \sqrt{\frac{k}{m}}$.

Example 1: A 300g mass on the end of a spring oscillates at 1.50Hz. Find (a)the angular frequency and (b)the spring constant.

In the upper image a mass, m, rests at the end of a spring of spring constant, k. In the middle image, the mass has been moved to the right a distance A. In the bottom image, the mass has been released and it is now at the position x with a speed v.



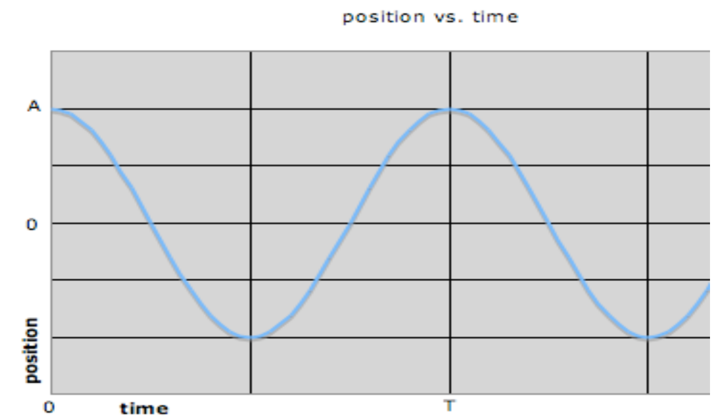
1. Draw the horizontal forces that act on the mass in the bottom image. Ignore friction.
2. Apply the Second Law and solve for the acceleration in terms of m, k, x, and A.

3. Sketch a graph of the position of the mass versus time. Indicate A in the graph.

4. The curve could be described mathematically as,

$$x = A\cos\theta$$

where x represents the (position) (time) and θ is related to the (position) (time).



5. In general, the maximum value of the cosine function is ___________. The maximum value of x is called the ______________________. So the A must represent the ____________________.

6. If the cos θ must be one at t = 0s, then the value of cos θ must be _____________ after one period. If θ is zero at t = 0s, then θ must be equal to _____________ after one period.

7. For any angle θ at any time t, complete the following ratio, $\frac{\theta}{2\pi} = \frac{\square}{T}$.

8. Solve this equation for θ then rewrite $x = A\cos\theta$ in terms of t:

Rewriting $x = A\cos 2\pi \frac{t}{T}$ in terms of $\omega \equiv \frac{2\pi}{T}$ gives the position as a function of time for the mass,

$$x = A\cos \omega t$$

9. Find the velocity as a function of time for the mass at the end of a spring,

10. Find the acceleration as a function of time for the mass at the end of a spring,

11. Substitute the equation for the position, $x = A\cos \omega t$, into the acceleration to get the acceleration as a function of position,

12. Compare the result of 11 to the result of part 2 to get the angular frequency for the mass on the end of a spring.

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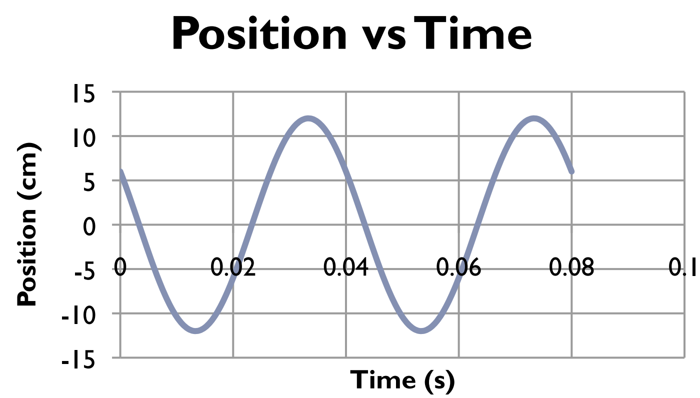
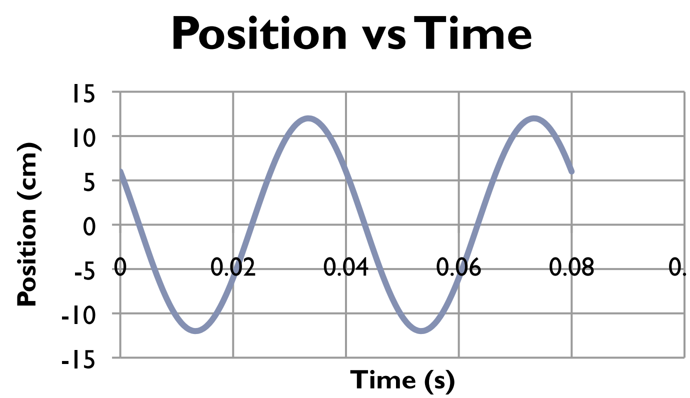
where A is the amplitude of the motion, ω is the angular frequency, and δ is the phase angle.

For a mass on the end of a spring, $\omega = \sqrt{\frac{k}{m}}$.

Example 2: From the graph at the right find (a)the amplitude, (b)the period, (c)the frequency, and (d)the angular frequency that describe the motion.



Example 3: From the graph at the right find (a)the phase angle, (b)the position as a function of time, (c)the velocity as a function of time, and (d)the acceleration as a function of time.



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