Systems Exhibiting SHM

Pre-Lecture Questions

Problem Set #36 (due next time)

Lecture Outline

1. The Torsional Pendulum
2. The Physical Pendulum
3. Other Forms of SHM

Pre-Class Summary:

An object with an acceleration that is equal to minus the product of some constant and the position is in SHM an obeys the SHM equations of motion,

$$a(x) = -\omega^2 x$$

$$v(x) = \pm\omega\sqrt{A^2 - x^2}$$

$$x(t) = A\cos(\omega t + \delta)$$

$$v(t) = -\omega A\sin(\omega t + \delta)$$

$$a(t) = -\omega^2 A\cos(\omega t + \delta)$$

The equations of motion for the x-component of uniform circular motion are identical to the equations of motion for SHM.

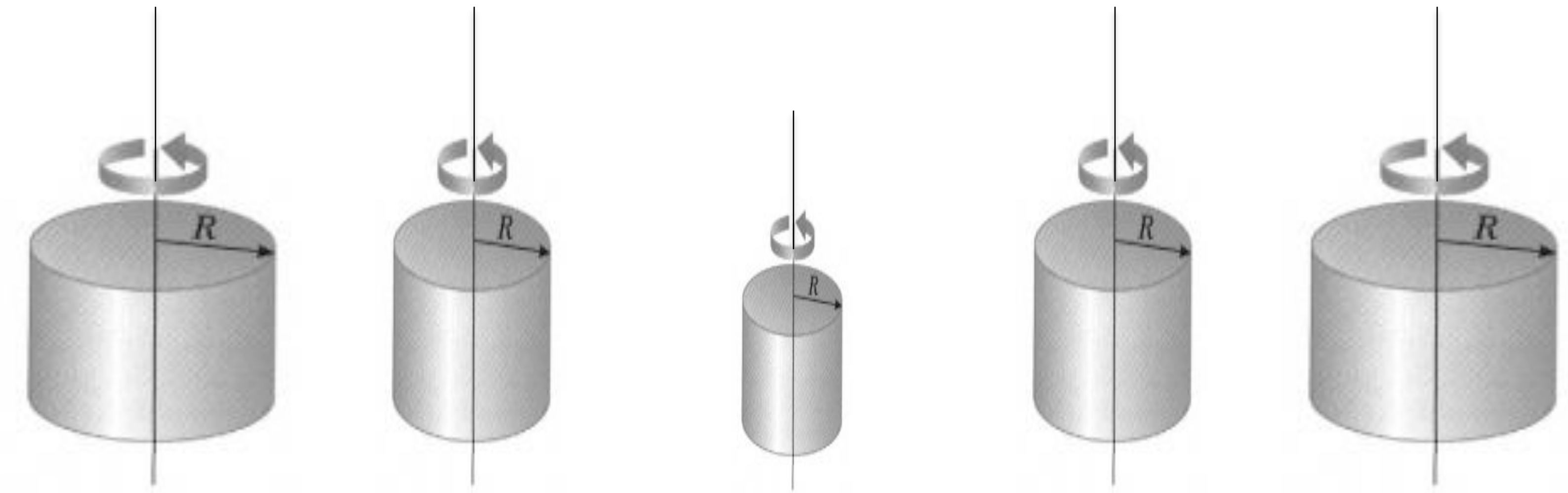
For a mass on the end of a spring, $\omega = \sqrt{\frac{k}{m}}$.

For a simple pendulum, $\omega = \sqrt{\frac{g}{\ell}}$.

For a torsional pendulum, $\omega = \sqrt{\frac{\kappa}{I}}$.

For a physical pendulum, $\omega = \sqrt{\frac{mgr}{I_p}}$.

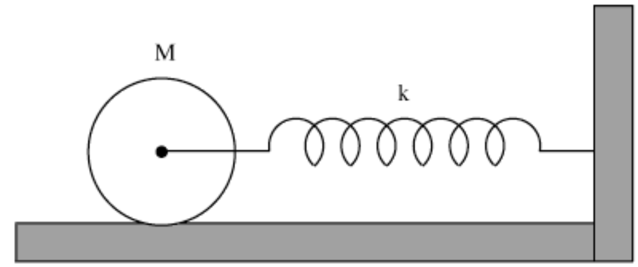
The five cylinders shown below are hung from identical strings. They have differing masses and radii. They are given a small twist and begin to oscillate back and forth. Rank them from greatest to least based upon the period of their oscillations.



A	B	C	D	E
M = 1kg	M = 4kg	M = 10kg	M = 5kg	M = 2kg
R = 10cm	R = 5cm	R = 1cm	R = 5cm	R = 10cm

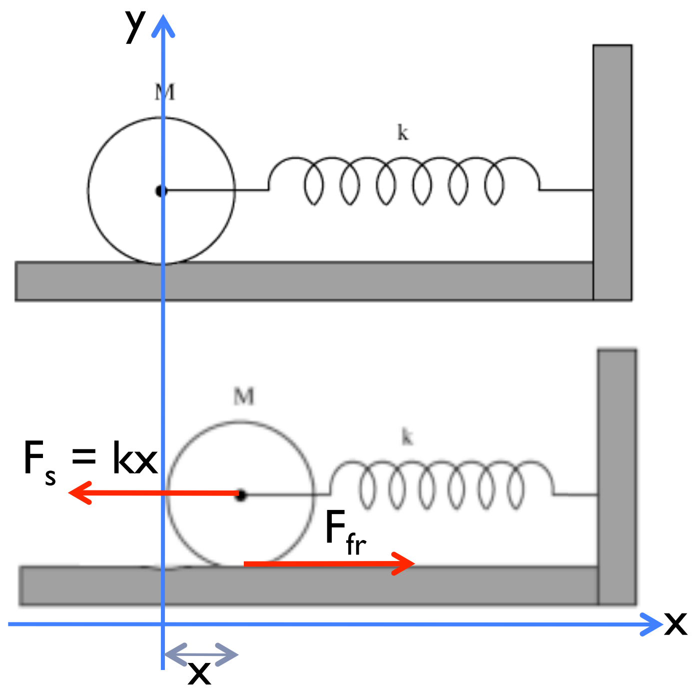
Example 1: Find the fraction that multiples ML^2 for the rotational inertia of a meter stick.

Example 2: A solid cylinder of mass M is attached to a horizontal massless spring with spring constant, k, so that it can roll without slipping along a horizontal surface, as shown. (a)Show that the motion is SHM and (b)find the angular frequency.



Moving to the right at equilibrium

Moving to the right slowing down



Applying the Second Laws,

$$\Sigma F = Ma \Rightarrow F_{fr} - kx = Ma_{cm} \qquad \Sigma\tau_{cm} = I\alpha_{cm} \Rightarrow F_{fr} r = \tfrac{1}{2} M r^2 \alpha_{cm} \Rightarrow F_{fr} = \tfrac{1}{2} M r \alpha_{cm}$$

Rolling without slipping means the cm acceleration is related to the angular acceleration – be careful about the minus sign.

$$a_{cm} = -r\alpha_{cm}$$

Lecture 36 - Summary

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