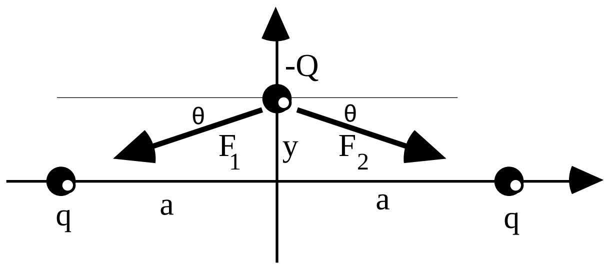
Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded. **For full credit you must explain clearly what you are doing especially if your solution involves symmetry arguments or uses Gauss's Law.**

1. The two charges q are each a distance a to the right and left of the origin as shown at the right. Find the magnitude and direction of the electric force on the charge -Q which is a very small distance y above the origin. Show that this force varies linearly with y.



The magnitudes of the two forces can be found with Coulomb's Rule,

$$\vec{F}_e = k\frac{q_1 q_2}{r^2}\hat{r} \qquad F_1 = F_2 = k\frac{qQ}{a^2 + y^2}$$

These must be added as vectors,

$F_x = F_2 \cos\theta - F_1 \cos\theta = 0$ as might be expected from symmetry considerations.

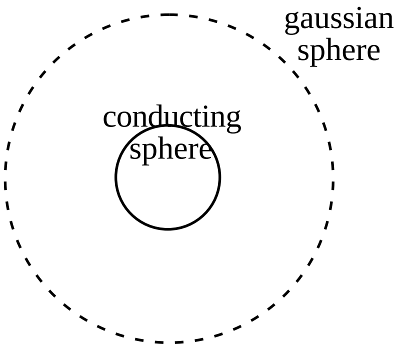
$$F_y = -F_2 \sin\theta - F_1 \sin\theta = -2k\frac{qQ}{a^2 + y^2}\sin\theta$$

Note that $\sin\theta = \frac{y}{\sqrt{a^2 + y^2}}$.

The net force is downward with a magnitude of $\boxed{F = 2k\frac{qQy}{\left(a^2 + y^2\right)^{\frac{3}{2}}}}$

This is not linear in y, but for y very small compared to a, the y in the denominator can be neglected giving $\boxed{F = 2k\frac{qQy}{a^3}}$ which is linear in y.

2. A solid metal sphere of radius 5.00cm creates an electric field of $2.25x10^6$N/C at a distance of 20.0cm from its center. Describe the charge distribution on the sphere.



Start by applying Gauss's Law, $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_o}$, to an imaginary sphere 20.0cm in radius centered on the conducting sphere. By the spherical symmetry of the problem, the field must be radial and constant on this imaginary sphere so,

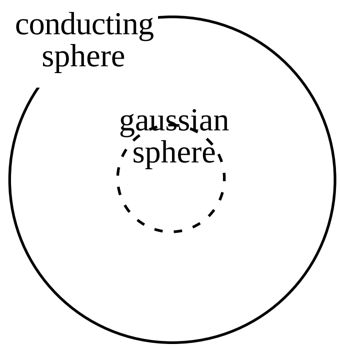
$$EA = \frac{q}{\varepsilon_o} \qquad q = \varepsilon_o EA = \varepsilon_o E 4\pi r^2$$

$$q = \left(8.85x10^{-12}\right)\left(2.25x10^6\right)4\pi\left(0.200\right)^2 = \underline{\underline{10.0\mu C}}$$

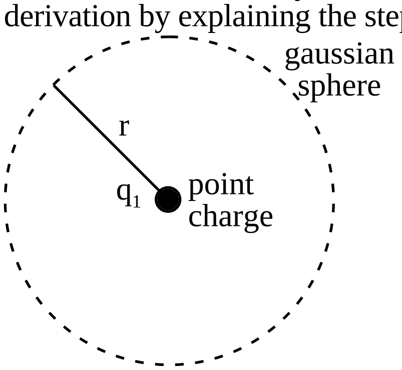
Using Gauss's Law and the same symmetry arguments for an imaginary sphere inside the metal sphere,

$$EA = \frac{q}{\varepsilon_o} \qquad q = \varepsilon_o EA\,.$$

However, the field inside a conductor is always zero, so the charge inside is also zero. Therefor, all the charge on the sphere is located on the surface.



3. The principle that the book calls "Coulomb's Law" I refer to as "Coulomb's Rule." I do this because laws cannot be derived while rules can be derived from laws and definitions. Coulomb’s Rule can be derived by starting with a point charge and applying Gauss's Law. Complete this derivation by explaining the steps carefully.



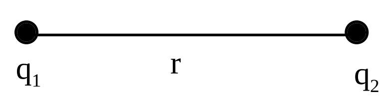
The field due to q_1 can be found by applying Gauss's Law,

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_o}\,,$$

to an imaginary sphere of radius r centered on the charge. By the spherical symmetry of the problem, the field must be radial and constant on this imaginary sphere so,

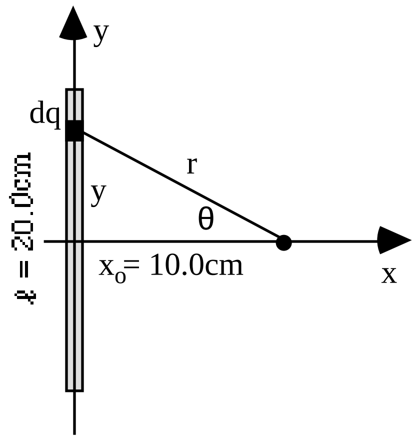
$$E4\pi r^2 = \frac{q_1}{\varepsilon_o} \qquad E = \frac{q_1}{4\pi\varepsilon_o r^2} = k\frac{q_1}{r^2}\,.$$

Now suppose a charge q_2 is in the field of q_1 a distance r away. The force on the q_2 can be found, by the definition of electric field.



$$E \equiv \frac{F}{q} \qquad k\frac{q_1}{r^2} = \frac{F}{q_2} \qquad F = k\frac{q_1 q_2}{r^2}$$

which is precisely Coulomb’s Rule.



4. A 20cm long wire has a uniform charge density, λ=3.00C/m. It is along the y-axis centered in the origin as shown. Find the electric potential at a distance x_o=10.0cm from the origin.

The potential due to the point charge dq is,

$$dV = k\frac{dq}{r}.$$

The charge can be written in terms of the charge density,
$dq = \lambda\, dy$ where $y = x_o \tan\theta$ so $dq = \lambda x_o \sec^2\theta d\theta$.

Since $\frac{x_o}{r} = \cos\theta \qquad r = x_o \sec\theta$.

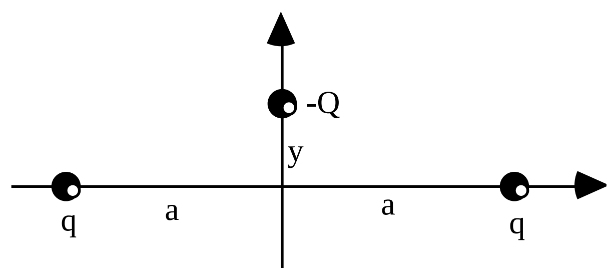
The potential can be written in terms of an integral over θ,

$$dV = k\frac{\lambda x_o \sec^2\theta d\theta}{x_o \sec\theta} = k\lambda \sec\theta d\theta \qquad V = k\lambda\int_{-\pi/4}^{\pi/4}\sec\theta d\theta$$

You will get full credit for getting this far, but since I have an integral table I'll complete the integration,

$$V = k\lambda\left[\ln\left(\sec\theta + \tan\theta\right)\right]_{-\pi/4}^{\pi/4} = k\lambda \ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$

5. The charge -Q shown at the right is a distance y (not very small) above the origin and moving downward. Find (a)the electric potential it feels, (b)its potential energy, (c)its potential energy when it reaches the origin. (d)Will it be moving faster or slower when it reaches the origin. Explain.



(a)Adding the potentials due to the point charges,

$$V = k\frac{q}{\sqrt{a^2+y^2}} + k\frac{q}{\sqrt{a^2+y^2}} = \frac{2kq}{\sqrt{a^2+y^2}}$$

(b)Using the definition of electric potential,

$$\Delta U \equiv q\Delta V = -Q\frac{2kq}{\sqrt{a^2+y^2}} = -\frac{2kqQ}{\sqrt{a^2+y^2}}.$$

(c)At the origin, y = 0 so,

$$\Delta U = -\frac{2kqQ}{a}$$

(d)Since the potential energy is now smaller (it is a bigger negative number) the kinetic energy must be larger by the Law of Conservation of Energy. Therefor, the charge must be moving faster.