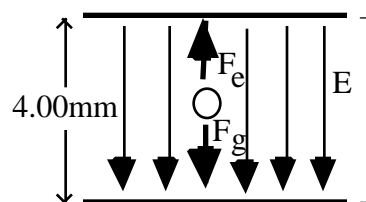


Name: _____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded. **For full credit you must explain clearly what you are doing especially if your solution involves symmetry arguments or uses Gauss's Law.**

1. A small plastic sphere of mass $6.00 \times 10^{-16} \text{ kg}$ is held motionless between two charged parallel plates 4.00 mm apart. Assume that the plastic sphere has three excess electrons on it. Find (a) the electric force on the sphere and (b) the electric field between the plates. (c) In the diagram at the right, show the polarity of the plates, sketch the electric field and show all the forces that act on the sphere.



(a) According to Newton's Second Law, $F = ma$ $F_e - F_g = 0$ $F_e = F_g$.

Putting in the numbers, $F_e = mg = (6.00 \times 10^{-16})(9.80) = \underline{\underline{5.88 \times 10^{-15} \text{ N}}}$.

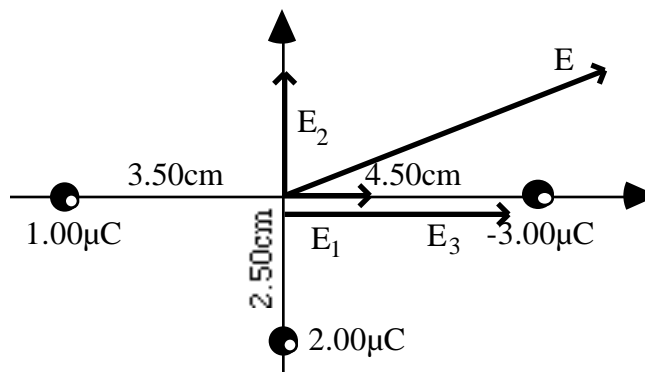
(b) Using the definition of electric field, $E = \frac{F}{q} = \frac{5.88 \times 10^{-15}}{3(1.60 \times 10^{-19})} = \underline{\underline{1.23 \times 10^4 \frac{\text{N}}{\text{C}}}}$.

2. For the three charges shown at the right, find the magnitude and direction of the electric field at the origin.

Using the field due to point charges,

$$\vec{E} = k \frac{q}{r^2} \hat{r}$$

the field due to each individual charge can be found.



$$E_1 = k \frac{q_1}{r_1^2} = (8.99 \times 10^9) \frac{1.00 \times 10^{-6}}{(0.0350)^2} = 7.34 \times 10^6 \frac{\text{N}}{\text{C}}$$

$$E_2 = k \frac{q_2}{r_2^2} = (8.99 \times 10^9) \frac{2.00 \times 10^{-6}}{(0.0250)^2} = 28.8 \times 10^6 \frac{\text{N}}{\text{C}}$$

$$E_3 = k \frac{q_3}{r_3^2} = (8.99 \times 10^9) \frac{3.00 \times 10^{-6}}{(0.0450)^2} = 13.3 \times 10^6 \frac{\text{N}}{\text{C}}$$

Adding the vector components,

$$E_x = E_1 + E_3 = 7.34 \times 10^6 + 13.3 \times 10^6 = 20.6 \times 10^6 \frac{\text{N}}{\text{C}} \quad \text{and} \quad E_y = E_2 = 28.8 \times 10^6 \frac{\text{N}}{\text{C}}.$$

Using the Pythagorean Theorem and the definition of the tangent,

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(20.6 \times 10^6)^2 + (28.8 \times 10^6)^2} = \underline{\underline{35.4 \times 10^6 \frac{\text{N}}{\text{C}}}}$$

$$= \arctan \frac{E_y}{E_x} = \arctan \frac{28.8}{20.6} = \underline{\underline{54.4^\circ}}$$

3. A thin conducting rod of length L has a uniform charge q distributed along it as shown at the right. Find the magnitude and direction of the electric field a distance y above the center. Remember that most of the credit is for setting the problem up.

By symmetry, the x -components will cancel to zero. The y -component is can be found using the field of the point charge dq ,

$$dE_y = dE \sin \theta = k \frac{dq}{r^2} \sin \theta.$$

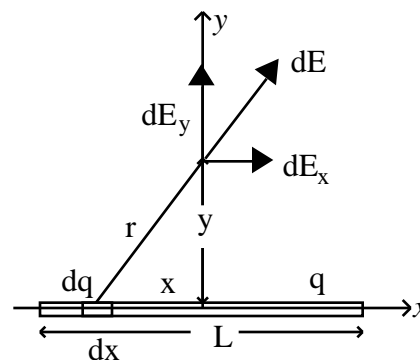
$$\text{Since the charge is uniform, } \frac{dq}{q} = \frac{dx}{L} \quad dq = \frac{q}{L} dx.$$

$$\text{Expressing all variables in terms of the angle, } r = y \csc \theta \quad \text{and} \quad x = -y \cot \theta \quad dx = y \csc^2 \theta d\theta.$$

$$\text{The } y\text{-component is now, } dE_y = k \frac{q}{L} \frac{y \csc^2 \theta d\theta}{y^2 \csc^2 \theta} \sin \theta = k \frac{q}{Ly} \sin \theta d\theta.$$

Adding the contributions from all the dq 's,

$$E_y = \int_0^\pi k \frac{q}{Ly} \sin \theta d\theta = -k \frac{q}{Ly} [\cos \theta]_0^\pi = 2k \frac{q}{Ly} \cos 0 = \underline{\underline{2k \frac{q}{Ly} \frac{L}{\sqrt{4y^2 + L^2}}}}.$$



4. For the charge distribution of problem 2, find the total electric flux that leaves an imaginary sphere centered at the origin that has a radius of (a) 2.00cm, (b) 3.00cm, (c) 4.00cm, and (d) 5.00cm

Gauss's Law states that the total flux that leaves any closed surface is equal to the enclosed charge divided by ϵ_0 . For each case we just need to find the enclosed charge and divide.

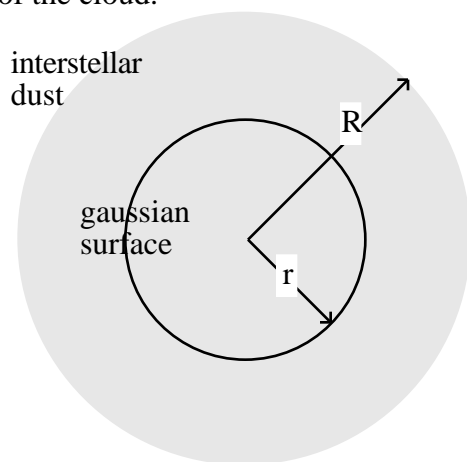
$$(a) q_{\text{enclosed}} = 0 = 0$$

$$(b) q_{\text{enclosed}} = 2.00\mu\text{C} = \frac{2.00 \times 10^{-6}}{8.85 \times 10^{-12}} = 2.26 \times 10^5 \frac{\text{Nm}^2}{\text{C}}$$

$$(c) q_{\text{enclosed}} = 2.00\mu\text{C} + 1.00\mu\text{C} = \frac{3.00 \times 10^{-6}}{8.85 \times 10^{-12}} = 3.39 \times 10^5 \frac{\text{Nm}^2}{\text{C}}$$

$$(d) q_{\text{enclosed}} = 2.00\mu\text{C} + 1.00\mu\text{C} - 3.00\mu\text{C} = 0 = 0$$

5. A spherical cloud of interstellar dust has a radius $4.00 \times 10^{10}\text{m}$ and an approximately uniform charge density of $3.00 \times 10^{-9} \frac{\text{C}}{\text{m}^3}$. Find the electric field at a distance of $2.00 \times 10^{10}\text{m}$ from the center of the cloud.



Since the charge density is uniformly distributed, the electric field must be constant on the surface of the gaussian sphere and it must point radially outward. The flux out of the sphere must be,

$$\oint \vec{E} \cdot d\vec{A} = E dA = E \oint dA = E 4\pi r^2.$$

The charge enclosed by the gaussian sphere is,

$$q_{\text{encl}} = \frac{4}{3} \pi r^3 \rho.$$

Applying Gauss's Law,

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\epsilon_0} \quad E 4\pi r^2 = \frac{\frac{4}{3} \pi r^3 \rho}{\epsilon_0} \quad E = \frac{\rho r}{3 \epsilon_0}.$$

Plugging in the numbers,

$$E = \frac{(3.00 \times 10^{-9})(2.00 \times 10^{10})}{3(8.85 \times 10^{-12})} = 2.26 \times 10^{12} \frac{\text{N}}{\text{C}}.$$