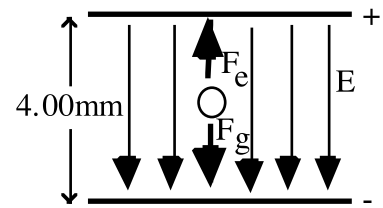
Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded. **For full credit you must explain clearly what you are doing especially if your solution involves symmetry arguments or uses Gauss's Law.**

1. A small plastic sphere of mass $6.00\text{x}10^{-16}$kg is held motionless between two charged parallel plates 4.00mm apart. Assume that the plastic sphere has three excess electrons on it. Find (a)the electric force on the sphere and (b)the electric field between the plates. (c)In the diagram at the right, show the polarity of the plates, sketch the electric field and show all the forces that act on the sphere.

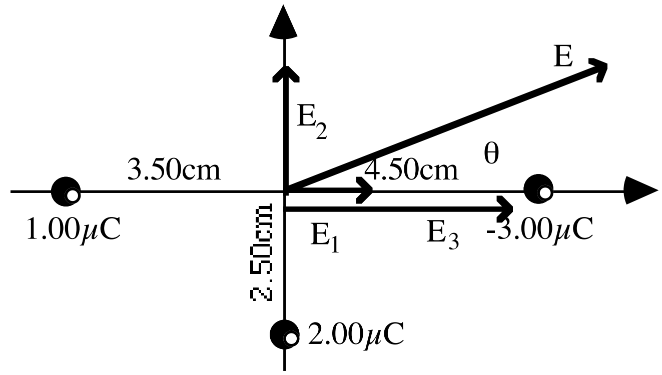


(a)According to Newton's Second Law, $\Sigma F = ma \Rightarrow F_e - F_g = 0 \Rightarrow F_e = F_g$.

Putting in the numbers, $F_e = mg = (6.00\text{x}10^{-16})(9.80) = \underline{\underline{5.88\text{x}10^{-15}\,\text{N}}}$.

(b)Using the definition of electric field, $E \equiv \frac{F}{q} = \frac{5.88\text{x}10^{-15}}{3(1.60\text{x}10^{-19})} = \underline{\underline{1.23\text{x}10^{4}\ \frac{\text{N}}{\text{C}}}}$.

2. For the three charges shown at the right, find the magnitude and direction of the electric field at the origin.



Using the field due to point charges,

$$\vec{E} = k\frac{q}{r^2}\hat{r}$$

the field due to each individual charge can be found.

$$E_1 = k\frac{q_1}{r_1^2} = (8.99\text{x}10^9)\frac{1.00\text{x}10^{-6}}{(0.0350)^2} = 7.34\text{x}10^6\,\tfrac{N}{C}$$

$$E_2 = k\frac{q_2}{r_2^2} = (8.99\text{x}10^9)\frac{2.00\text{x}10^{-6}}{(0.0250)^2} = 28.8\text{x}10^6\,\tfrac{N}{C}$$

$$E_3 = k\frac{q_3}{r_3^2} = (8.99\text{x}10^9)\frac{3.00\text{x}10^{-6}}{(0.0450)^2} = 13.3\text{x}10^6\,\tfrac{N}{C}$$

Adding the vector components,

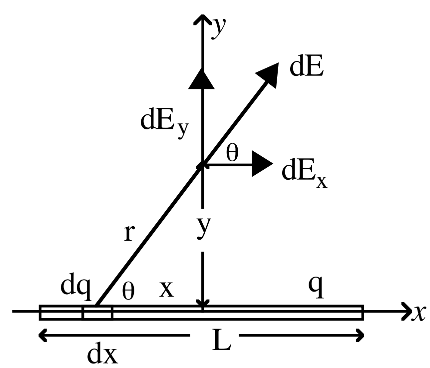
$E_x = E_1 + E_3 = 7.34\text{x}10^6 + 13.3\text{x}10^6 = 20.6\text{x}10^6\,\tfrac{N}{C}$ and $E_y = E_2 = 28.8\text{x}10^6\,\tfrac{N}{C}$.

Using the Pythagorean Theorem and the definition of the tangent,

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\left(20.6\text{x}10^6\right)^2 + \left(28.8\text{x}10^6\right)^2} = \underline{\underline{35.4\text{x}10^6\,\tfrac{N}{C}}}$$

$$\theta = \arctan\frac{E_y}{E_x} = \arctan\frac{28.8}{20.6} = \underline{\underline{54.4^\circ}}$$

3. A thin conducting rod of length L has a uniform charge q distributed along it as shown at the right. Find the magnitude and direction of the electric field a distance y above the center. Remember that most of the credit is for setting the problem up.



By symmetry, the x-components will cancel to zero. The y-component is can be found using the field of the point charge dq,

$$dE_y = dE\sin\theta = k\frac{dq}{r^2}\sin\theta.$$

Since the charge is uniform, $\frac{dq}{q} = \frac{dx}{L} \Rightarrow dq = \frac{q}{L}dx$.

Expressing all variables in terms of the angle, $r = y\csc\theta$ and $x = -y\cot\theta \Rightarrow dx = y\csc^2\theta d\theta$.

The y-component is now, $dE_y = k\frac{q}{L}\frac{y\csc^2\theta d\theta}{y^2\csc^2\theta}\sin\theta = -k\frac{q}{Ly}\sin\theta d\theta$.

Adding the contributions from all the dq's,

$$E_y = \int_{\theta_o}^{\pi-\theta_o} k\frac{q}{Ly}\sin\theta d\theta = -k\frac{q}{Ly}\left[\cos\theta\right]_{\theta_o}^{\pi-\theta_o} = 2k\frac{q}{Ly}\cos\theta_o = \underline{\underline{2k\frac{q}{Ly}\cdot\frac{L}{\sqrt{4y^2+L^2}}}}.$$

4. For the charge distribution of problem 2, find the total electric flux that leaves an imaginary sphere centered at the origin that has a radius of (a)2.00cm, (b)3.00cm, (c)4.00cm, and (d)5.00cm

Gauss's Law states that the total flux that leaves any closed surface is equal to the enclosed charge divided by ε_o. For each case we just need to find the enclosed charge and divide.

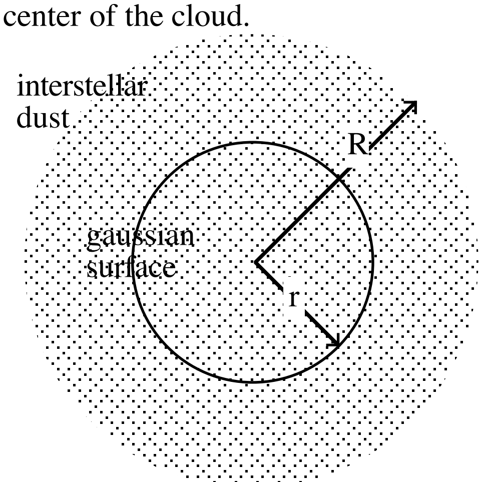
(a) $q_{enclosed} = 0 \Rightarrow \Phi = 0$

(b) $q_{enclosed} = 2.00\mu C \Rightarrow \Phi = \dfrac{2.00x10^{-6}}{8.85x10^{-12}} = 2.26x10^5 \frac{Nm^2}{C}$

(c) $q_{enclosed} = 2.00\mu C + 1.00\mu C \Rightarrow \Phi = \dfrac{3.00x10^{-6}}{8.85x10^{-12}} = 3.39x10^5 \frac{Nm^2}{C}$

(d) $q_{enclosed} = 2.00\mu C + 1.00\mu C - 3.00\mu C = 0 \Rightarrow \Phi = 0$

5. A spherical cloud of interstellar dust has a radius $4.00x10^{10}$m and an approximately uniform charge density of $3.00x10^{-9}\frac{C}{m^3}$. Find the electric field at a distance of $2.00x10^{10}$m from the center of the cloud.



Since the charge density is uniformly distributed, the electric field must be constant on the surface of the gaussian sphere and it must point radially outward. The flux out of the sphere must be,

$$\oint \vec{E} \bullet d\vec{A} = \int EdA = E\int dA = E4\pi r^2.$$

The charge enclosed by the gaussian sphere is,

$$q_{encl} = \rho \tfrac{4}{3}\pi r^3.$$

Applying Gauss's Law,

$$\oint \vec{E} \bullet d\vec{A} = \frac{q_{encl}}{\varepsilon_o} \Rightarrow E4\pi r^2 = \frac{4\rho\pi r^3}{3\varepsilon_o} \Rightarrow E = \frac{\rho r}{3\varepsilon_o}.$$

Plugging in the numbers,

$$E = \frac{(3.00x10^{-9})(2.00x10^{10})}{3(8.85x10^{-12})} = 2.26x10^{12}\,\tfrac{N}{C}.$$