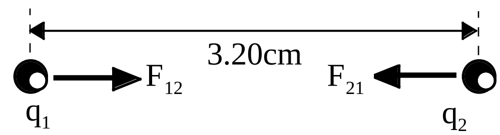
Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded. **For full credit you must explain clearly what you are doing especially if your solution involves symmetry arguments or uses Gauss's Law.**

1. The charge q_1=+12.0μC has a mass of 3.00g and the charge q_2=-8.0μC has a mass of 5.00g. At some instant in time they are a distance 3.20cm apart as shown at the right. Find (a)the magnitude and direction of the electric force on the charge q_1, (b)the magnitude and direction of the electric force on the charge q_2, (c)the magnitude and direction of the acceleration of the charge q_1 and (d)the magnitude and direction of the acceleration of the charge q_2. Ignore gravity.



(a)The force is given by Coulomb's Rule,

$$F_{12} = k\frac{q_1 q_2}{r^2} = (8.99x10^9)\frac{(12.0x10^{-6})(8.0x10^{-6})}{(0.0320)^2} = \underline{\underline{843N}}.$$

Since the charges are opposite, the forces are attractive. Therefor, the force on q_1 is to the right.

(b)The force that q_2 exerts on q_1 must be equal and opposite to the force the q_1 exerts on q_2 by Newton's Third Law. Therefor, F_{21}=843N to the left.

(c)From Newton's Second Law,

$$\Sigma F = ma \qquad F_{12} = m_1 a \qquad a = \frac{F_{12}}{m_1} = \frac{843}{0.00300} = \underline{\underline{2.81x10^5\ m/s^2}},$$

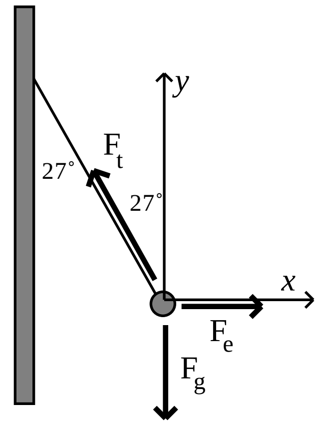
in the direction of the force (to the right).

(d)Again, using Newton's Second Law,

$$\Sigma F = ma \qquad F_{21} = m_2 a \qquad a = \frac{F_{21}}{m_2} = \frac{843}{0.00500} = \underline{\underline{1.69x10^5\ m/s^2}},$$

in the direction of the force (to the left).

2. A small ball has a charge of 5.00μC and mass 1.50g It hangs from a 20.0cm long thread that is connected to a very large charged metal plate as shown at the right. The thread makes a 27° angle with the plate. Find the electric field produced by the plate at the location of the ball.



The forces on the ball are shown at the right. Using Newton's Second Law,

$$\Sigma F_x = ma_x \qquad F_e - F_t \sin 27^\circ = 0 \qquad F_e = F_t \sin 27^\circ \text{ and}$$
$$\Sigma F_y = ma_y \qquad F_t \cos 27^\circ - F_g = 0 \qquad F_g = F_t \cos 27^\circ .$$

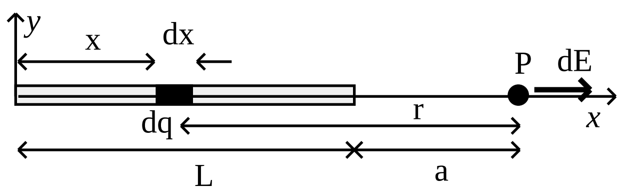
Dividing the first equation by the second eliminates the tension,

$$\frac{F_e}{F_g} = \frac{F_t \sin 27^\circ}{F_t \cos 27^\circ} = \tan 27^\circ \qquad F_e = F_g \tan 27^\circ .$$

Using the mass/weight rule and the definition of electric field,

$$qE = mg\tan 27^\circ \qquad E = \frac{mg}{q}\tan 27^\circ = \frac{(0.00150)(9.80)}{5.00\text{x}10^{-6}}\tan 27^\circ = \underline{\underline{1500\text{N/C}}} .$$

3. A rod of length L has a uniformly distributed charge Q. Find the field at the point P a distance a away from the end of the rod as shown at the right.



The rod can be broken up into a collection of point charges, dq. The field due to the point charges is,

$$dE = k\frac{dq}{r^2}$$

Since the cahrge is uniformly distributed, $\frac{dq}{Q} = \frac{dx}{L} \qquad dq = \frac{Q}{L}dx .$

The distance from the point charge to the point P is, $x + r = L + a \qquad r = L + a - x$.

Substituting,

$$dE = k\frac{Q}{L}\frac{dx}{(L + a - x)^2} .$$

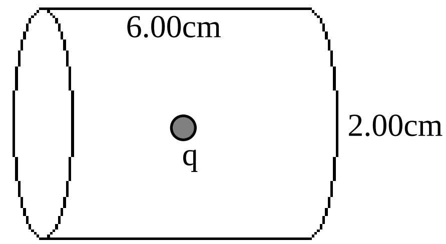
Summing the fields due to all the dq's,

$$\int dE = \int k\frac{Q}{L}\frac{dx}{(L + a - x)^2} \qquad E = k\frac{Q}{L}\int_0^L \frac{dx}{(L + a - x)^2} .$$

Let $u = L + a - x \qquad du = -dx$ and the integral becomes,

$$E = k\frac{Q}{L}\int_a^{L+a} u^{-2}du = k\frac{Q}{L}\left[-\frac{1}{L + a} + \frac{1}{a}\right] \qquad \underline{\underline{E = k\frac{Q}{a(L + a)}}} .$$

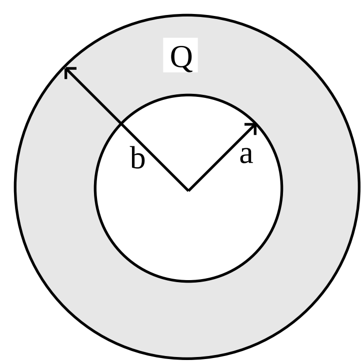
4. A point charge of 75.0μC is placed at the center of a cylindrical Gaussian surface 6.00cm long and 2.00cm in diameter. Find the total flux that leaves the cylinder.



According to Gauss's Law, the net flux leaving a volume equals the enclosed charge divided by ε_o.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{75.0x10^{-6}}{8.85x10^{-12}} = 8.47x10^{6}\ \frac{N\,m^2}{C}\ .$$

5. A spherical conducting shell of inner radius a and outer radius b has a net charge Q distributed on it. Find the electric field as a function of the distance from the center r for (a)r<a, (b)a<r<b, (c)r>b and (d)explain where the charge Q will be found (be very specific).



The charge will distribute itself in a spherically symmetric fashion. Therefor, the field will be spherically symmetric also. A Gaussian sphere of any radius r will then have a constant field at its surface and this field will be perpendicular to the surface at any point. Applying Gauss's Law with these symmetry conditions,

$$\oint \vec{E}\cdot d\vec{A} = \oint EdA = EA = E4\pi r^2 = \frac{q_{enclosed}}{\varepsilon_o} \qquad E = k\frac{q_{enclosed}}{r^2}\ .$$

(a)For $r<a$ there is no charge enclosed so E=0.
(b)For $a<r<b$ E=0 because it is inside a conducting material.
This has nothing to do with Gauss's Law.
(c)For $r>b$ the charge enclosed is Q so, $E = k\frac{Q}{r^2}$.
(d)For all Gaussian spheres of $r<b$ the field is zero therefor no charge can be enclosed.
The charge Q must therefor lie on the outer surface of the conducting sphere at r=b.