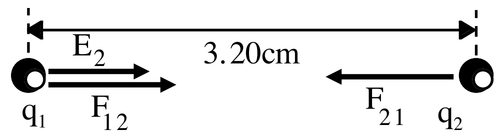
Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded. **For full credit you must explain clearly what you are doing especially if your solution involves symmetry arguments or uses Gauss's Law.**

1. The charge q_1=+12.0μC has a mass of 3.00g and the charge q_2=-8.0μC has a mass of 5.00g. At some instant in time they are a distance 3.20cm apart as shown at the right. Find (a)the electric field felt by the charge q_1, (b)the electric force on the charge q_1, (c)the electric force on the charge q_2 and (d)sketch the directions of these vectors in the drawing at the right.



(a)The field felt by q_1 is created by the point charge q_2,

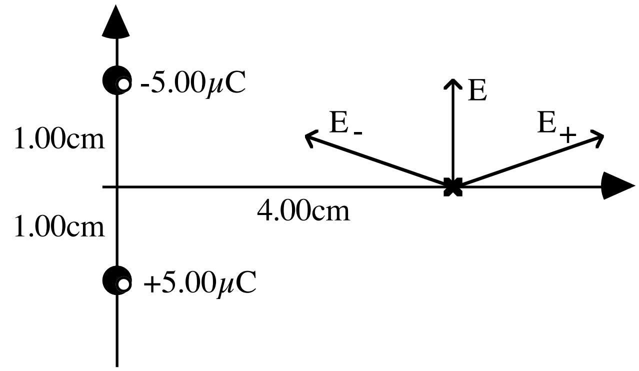
$$E_2 = k\frac{q_2}{r^2} = (9.00\text{x}10^9)\frac{8.00\text{x}10^{-6}}{(0.0320)^2} \Rightarrow \boxed{E_2 = 7.03\text{x}10^7 \text{N/C}}$$

(b)The force on q_1 can be found from the definition of electric field,

$$F_{12} = q_1E_2 = (12.0\text{x}10^{-6})(7.03\text{x}10^7) \Rightarrow \boxed{F_{12} = 844\text{N}}$$

(c) The force on q_2 is equal and opposite to the force on q_1 according to Newton's Third Law, $\boxed{F_{21} = 844\text{N}}$

2. The dipole shown at the right consists of two equal but opposite 5.00μC charges separated by 2.00cm. Find the electric field 4.00cm from the center of the dipole perpendicular to the dipole axis.



The field due to a point charge the point charges are,

$$\vec{E} = k\frac{q}{r^2}\hat{r} \Rightarrow E_+ = E_- = k\frac{q}{r^2}$$

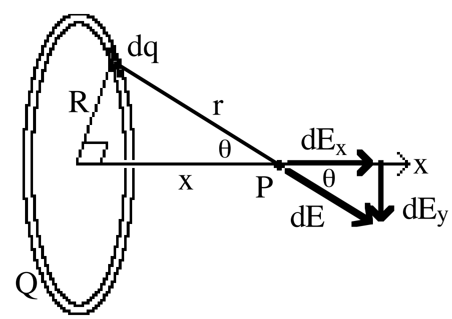
The horizontal components will cancel leaving only the vertical components,

$$E = E_+ \sin\theta + E_- \sin\theta = 2k\frac{q}{r^2}\sin\theta$$

where $r = \sqrt{4^2 + 1^2} = 4.12\text{cm}$ and $\sin\theta = \frac{1}{4.12} = 0.243$. Finally,

$$\boxed{E = 2(9\text{x}10^9)\frac{5\text{x}10^{-6}}{0.0412^2}(0.243) = 1.29\text{x}10^7 \text{ N / C}}$$

3. Find the field due to a ring of total charge Q and radius R at the point P a distance x from the center of the ring along the axis as shown at the right.



The field due to the point charge dq is,

$$dE = k\frac{dq}{r^2}.$$

By the symmetry of the ring, the y-components will cancel leaving only the x-components to add up.

$$dE_x = dE\cos\theta = k\frac{dq}{r^2}\cos\theta \Rightarrow E_x = \int k\frac{dq}{r^2}\cos\theta.$$

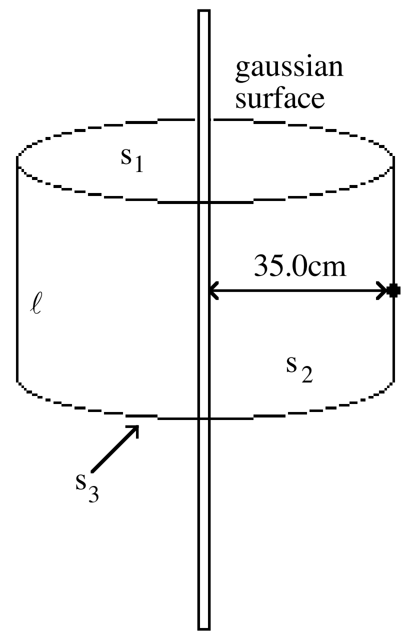
For all the dq's the r is constant and so is the angle θ. Therefor,

$$E_x = \frac{k}{r^2}\cos\theta\int dq.$$

The sum of all the dq's is just Q. Therefor $E_x = k\frac{Q}{r^2}\cos\theta$.

Writing r and cosθ in terms of R and x gives the answer $\boxed{E_x = k\frac{Qx}{\left(R^2 + x^2\right)^{3/2}}}$.

4. A very long thin line of uniformly distributed charge has $250\mu C$ per meter of length. Find the electric field 35.0cm away.



By symmetry the field points radially away from the line. Therefor, choose a cylinder of radius r=35.0cm for the gaussian surface and apply Gausses Law,

$$\oint \vec{E} \bullet d\vec{A} = \int_{s_1} \vec{E} \bullet d\vec{A} + \int_{s_2} \vec{E} \bullet d\vec{A} + \int_{s_3} \vec{E} \bullet d\vec{A} = \frac{q_{encl}}{\varepsilon_o}$$

Since the field is radial, no flux leaves s_1 or s_3. So,

$$\int_{s_2} \vec{E} \bullet d\vec{A} = \frac{q_{encl}}{\varepsilon_o}$$

On the surface s_2 the field must be constant by symmetry. Further, it points parallel to each area element. So,

$$\int_{s_2} \vec{E} \bullet d\vec{A} = \int_{s_2} E dA = E\int_{s_2} dA = E2\pi r\ell = \frac{q_{encl}}{\varepsilon_o}$$

The charge enclosed is the charge density times the length,

$$E2\pi r\ell = \frac{\lambda\ell}{\varepsilon_o} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_o r} = \frac{2k\lambda}{r} = \frac{2(9.00\text{x}10^9)(250\text{x}10^{-6})}{0.350}$$

Finally, $\boxed{E = 1.29\text{x}10^7 \frac{N}{C}}$

5. Consider a non-conducting cube centered at the origin. The length of each side is d and it contains a uniformly distributed charge Q throughout its volume. (a)Can you use Gauss's Law to find the total electric flux that leaves the cube? If so, find the result. If not, explain why not. (b)Can you use Gauss's Law to find the electric field at a distance r from the origin? If so, find the result. If not, explain why not.

(a)Yes! Gauss's Law states that the total flux that leaves any volume equals the enclosed charge over ε_o:

$$\boxed{\Phi = \frac{q_{encl}}{\varepsilon_o} = \frac{Q}{\varepsilon_o}}$$

(b)To use Gauss's Law to find an electric field a large degree of symmetry must exist. This problem doesn't have enough symmetry to find the field.