Physics 4B SECOND EXAM Chapters 26 - 28 Spring 1995

Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You <u>must</u> show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. The plates of a parallel plate capacitor are 4.00cm apart. The capacitor is charged when it is connected to a battery. The battery is disconnected then the separation between the plates is changed until the energy stored in the capacitor is doubled. Find the new separation between the plates.

The energy of a capacitor is, $U = \frac{1}{2}\frac{Q^2}{C}$. Using the capacitance of parallel plates $C = \varepsilon_o \frac{A}{d}$,

$$U = \frac{1}{2}\frac{Q^2}{\varepsilon_o \frac{A}{d}} = \frac{Q^2 d}{2\varepsilon_o A}.$$

Since the battery is disconnected, the charge on the plates remains fixed. The new energy is,

$$U' = \frac{Q^2 d'}{2\varepsilon_o A}.$$

The new energy is twice the original energy so,

$$U' = 2U \qquad \frac{Q^2 d'}{2\varepsilon_o A} = 2\frac{Q^2 d}{2\varepsilon_o A} \qquad d' = 2d = \underline{\underline{8.00\text{cm}}}$$

2. A capacitor C is charged to an initial voltage V_o. Then it is connected to a resistor R and it begins to discharge. (a)Write down (don't derive) the equation for the charge on this capacitor as a function of time. (b)Find an expression for the current through the resistor as a function of time. (c)Find the total charge that passes through the resistor from the expression for the current. (d)Explain why the answer you get "makes sense."

(a) For a discharging capacitor, $q = CV_o e^{-t/RC}$.

(b) Using the definition of current, $I \equiv \frac{dQ}{dt}$,

$$I = \frac{d}{dt}\left(CV_o e^{-t/RC}\right) = CV_o\left(-\frac{1}{RC}\right)e^{-t/RC} \qquad \underline{\underline{-\frac{V_o}{R}e^{-t/RC}}}$$

(c) Again, using the definition of current, $I \equiv \frac{dQ}{dt}$,

$$\int_0^Q dQ = \int_0^\infty I dt \qquad Q = \int_0^\infty -\frac{V_o}{R}e^{-t/RC}dt \qquad Q = -\frac{V_o}{R}\int_0^\infty e^{-t/RC}dt$$

To do the integral, let $u = -t/RC \qquad du = -\frac{1}{RC}dt$.

$$Q = -\frac{V_o}{R}(-RC)\int_0^{-\infty} e^u du = CV_o\left(e^{-\infty} - e^0\right) \qquad \underline{\underline{Q = -CV_o}}$$

(d)The total charge that passes through the resistor must equal the total charge that the capacitor had to start with!

3. A precision 1.00Ω resistor is made from 0.100mm diameter copper wire. Find the length of the wire needed.

Use the definition of resistance, $R = \rho\frac{\ell}{A}$.

Solve for the length, $\ell = \frac{RA}{\rho} = \frac{R\pi D^2}{4\rho}$.

Plugging in the numbers, $\ell = \frac{(1.00)\pi(10^{-4})^2}{4(1.7x10^{-8})} = \underline{\underline{46.2cm}}$

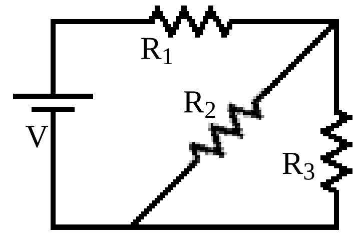
4. Use an analogy between traffic on a highway and an electrical circuit to describe charge, current and Kirchoff's Junction Theorem. Be sure to state the units for "charge" and "current."

Each car could represent a charge moving in the circuit. The units of charge would be "cars."

The current is the rate of charge flow. In this example, it would be the number of cars that pass a point per second. The units would be "cars/second."

The Junction Theorem states that the current flowing into any junction must equal the current leaving the junction. The equivalent statement here is that the number of cars entering an intersection must equal the number that leave.

5. In the circuit shown, the current through R_3 is measured to be 4.00mA. Find (a)the voltage across each resistor in the circuit, (b)the current through the other two resistors, (c)the current supplied by the battery, and (d)the voltage of the battery. R_1=1.00kΩ, R_2=3.00kΩ and R_3=6.00kΩ.



V(V)	I(mA)	R(kΩ)
12.0	12.0	1.00
24.0	8.00	3.00
24.0	4.00	6.00
battery 36.0	battery 12.0	battery XXX

The voltage across R_3 can be found using Ohm's Rule, $V = IR = (4)(6) = 24V$.
Since R_2 and R_3 are in parallel the loop theorem requires that they have the same voltage drop.
Now Ohm's Rule can be used to find the current through R_2, $I = \frac{V}{R} = \frac{24}{3} = 8.00mA$.
By the junction theorem, the current through R_1 must equal the sum of the currents through R_2 and R_3.
$I_1 = I_2 + I_3 = 8 + 4 = 12mA$.

The voltage across R_1 can be found from Ohm's Rule, $V = IR = (12)(1) = 12V$.
The battery is in series with R_1 so it must have the same current.
The voltage of the battery must equal the voltage across R_1 plus either the voltage across R_2 or R_3 by the loop theorem. $V = V_1 + V_3 = 12 + 24 = 36V$.