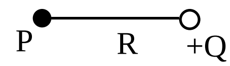
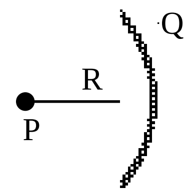
Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

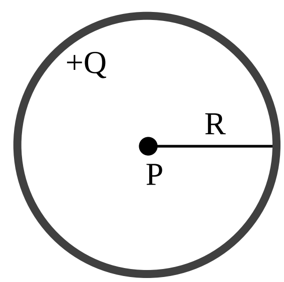
1. Each of the three charge distributions (point charge, arc, and ring) at the right have the same total charge Q. Find the potential at the point P for the (a)point charge, (b)arc and (c)ring. (d)Rank the distributions according to the strength of the electric field at the point P. Be sure to explain your reasoning for full credit.



point charge



arc of charge



ring of charge

(a)The potential due to the point charge is $\boxed{V = k\frac{Q}{R}}$.

(b)The potential due to the arc can be found by breaking the arc into small point charges dq, but since each dq is the same distance R away, the potential is the same as for the point charge Q,

$$V = \int k\frac{dq}{R} = k\frac{1}{R}\int dq \qquad \boxed{V = k\frac{Q}{R}}$$

(c)The potential due to the ring is also the same as the potential due to the point charge Q for the same reason as in part (b),

$$\boxed{V = k\frac{Q}{R}}$$

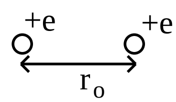
(d)The field due to the point charge Q is, $E_{point} = k\frac{Q}{R^2}$.

The field at the center of the ring is zero by the symmetry, $E_{ring} = 0$.

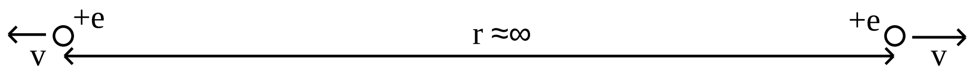
The field due to the arc can be found by breaking the arc into small point charges dq. By the symmetry, the vertical components will cancel to zero leaving only the horizontal components. When the horizontal components are added, they will sum to something less than the sum of all the magnitudes since some of the components are smaller than the magnitude.

Therefor, in rank order, $\boxed{E_{ring} < E_{arc} < E_{point}}$.

2. Two protons are initially at rest 2.00nm apart. The electric force causes them to separate. Find their speeds when they are very far apart.



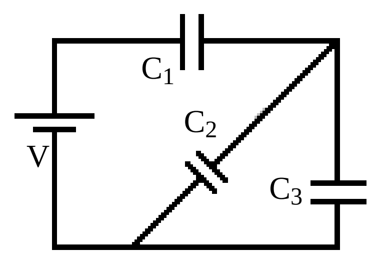
Initially the energy of the system is all electrical potential energy, $\Delta U = q\Delta V \qquad \mathcal{E} = ek\frac{e}{r_o} = k\frac{e^2}{r_o}$.



Afterward the potential energy is completely converted to kinetic energy, $\mathcal{E} = K = 2\left(\frac{1}{2}mv^2\right) = mv^2$.
Applying the Law of Conservation of Energy,

$$k\frac{e^2}{r_o} = mv^2 \qquad v = \sqrt{\frac{ke^2}{mr_o}} = \sqrt{\frac{(8.99x10^9)(1.60x10^{-19})^2}{(1.67x10^{-27})(2.00x10^{-9})}} \qquad \boxed{v = 8.30x10^3\ m/s}$$

3. The battery shown in the circuit at the right has a potential difference of 18.0V. Find (a)the potential difference across each capacitor and (b)the charge on each capacitor (C_1=9.00μF, C_2=6.00μF and C_3=12.0μF).



Q(μC)	C(μF)	V(V)
108	9.00	12.0
36.0	6.00	6.00
72.0	12.0	6.00

C_2 and C_3 are in parallel,
$C_p = C_2 + C_3 = 6 + 12 = 18.0\mu F$
This equivalent capacitance is in series with C_1,

$$\frac{1}{C} = \frac{1}{C_p} + \frac{1}{C_1} = \frac{1}{18} + \frac{1}{9} = \frac{3}{18} \qquad C = 6.00\mu F$$

The total charge supplied by the battery can be found using the definition of capacitance,
$Q = CV = (6.00\mu F)(18.0V) = 108\mu C$.
This charge must appear on C_1 because it is in series with the battery, $V_1 = \frac{Q}{C_1} = \frac{108}{9} = 12V$.

By the loop theorem, this leaves 18.0-12.0=6.00V on C_2 and C_3.
The charge on each must be,
$Q_2 = C_2V_2 = (6.00\mu F)(6.00V) = 36.0\mu C$
$Q_3 = C_3V_3 = (12.0\mu F)(6.00V) = 72.0\mu C$.

4. A wire made of a new material is 4.00m long and 6.00mm in diameter. It has a resistance of 15.0mΩ. A potential difference of 23.0V is applied. Find (a)the current through the wire and (b)the resistivity of the material.

(a)Using Ohm's Rule $V = IR \quad I = \frac{V}{R} = \frac{23.0}{15.0m}$ $\boxed{I = 1.53kA}$

(b)Using the definition of resistance,

$$R \equiv \rho\frac{\ell}{A} \quad \rho = \frac{RA}{\ell} = \frac{R\pi D^2}{4\ell} = \frac{(0.0150)\pi(0.00600)^2}{4(4.00)} \quad \boxed{\rho = 1.06x10^{-7}\Omega\ m}$$

5. A 1.50V dry cell battery is connected to an initially uncharged 16,000μF capacitor. It takes 200ms for the capacitor to acquire a charge of 18,000μC. Find the internal resistance of the battery.

For a charging capacitor, $q = CV\left(1 - e^{-t/RC}\right)$. The resistance here is the internal resistance of the battery. Solving for the internal resistance,

$$R = -\frac{t}{C\ln\left(1 - \frac{q}{CV}\right)} = -\frac{0.200}{(0.016)\ln\left(1 - \frac{18.0}{(16.0)(1.5)}\right)} \quad \boxed{R = 9.02\Omega}$$