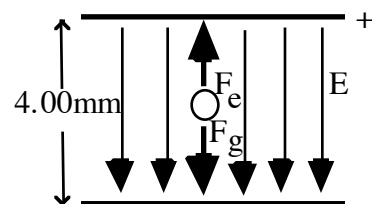


Name: \_\_\_\_\_

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A small plastic sphere of mass  $5.00 \times 10^{-16} \text{ kg}$  is held motionless between two charged parallel plates  $4.00 \text{ mm}$  apart. Assume that the plastic sphere has two excess electrons on it. Find (a) the electric force on the sphere, (b) the electric field between the plates and (c) the potential difference between the plates. (d) In the diagram at the right, show the polarity of the plates, sketch the electric field and show all the forces that act on the sphere.



(a) According to Newton's Second Law,  $\Sigma F = ma \Rightarrow F_e - F_g = 0 \Rightarrow F_e = F_g$ .

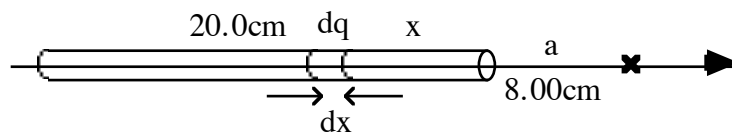
Putting in the numbers,  $F_e = mg = (5.00 \times 10^{-16})(9.80) = \underline{4.90 \times 10^{-15} \text{ N}}$ .

(b) Using the definition of electric field,  $E = \frac{F}{q} = \frac{4.90 \times 10^{-15}}{2(1.60 \times 10^{-19})} = \underline{1.53 \times 10^4 \frac{\text{V}}{\text{m}}}$ .

(c) The electric potential is  $\Delta V = -\int \vec{E} \cdot d\vec{s}$ .

Since the field between the plates is constant,  $V = Ed = (1.53 \times 10^4)(4.00 \times 10^{-3}) = \underline{61.3 \text{ V}}$ .

2. Find the potential due to a 20.0cm long rod that has a linear charge density of  $2.00\mu\text{C}/\text{m}$  at a point 8.00cm from the end of the rod along the same axis as the rod.



The potential due to the point charge dq is,  $dV = k \frac{dq}{r}$ .

The charge dq can be written in terms of the charge density,  $dq = \lambda dx$ .

The distance between dq and the point where we need the field is,  $r = x + a$ .

The potential is,  $dV = k \frac{\lambda dx}{x + a} \Rightarrow V = k\lambda \int_0^L \frac{dx}{x + a} = k\lambda \ln\left(\frac{L + a}{a}\right)$ .

Plugging in the numbers,  $V = (9 \times 10^9)(2 \times 10^{-6}) \ln\left(\frac{28}{8}\right) = \underline{\underline{2.25 \times 10^4 \text{ V}}}$ .

3. An automobile battery has a potential difference of 12.0V and sends current through a circuit of total resistance  $1.50\Omega$  that contains a copper wire 1.00m long with an  $0.300\text{cm}^2$  cross-sectional area. Find (a) the current through the wire, (b) the energy lost to heat in the circuit in one hour and (c) the distance traveled by an electron in the circuit in an hour.

(a) Using Ohm's Rule,  $V = IR \Rightarrow I = \frac{V}{R} = \frac{12.0}{1.50} = \underline{\underline{8.00\text{A}}}$ .

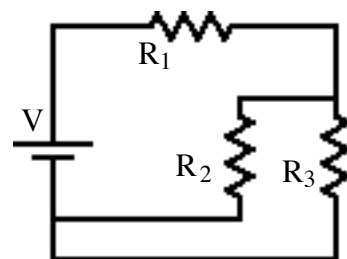
(b) Using the definition of power and the power consumed in a resistor,

$P = \frac{\Delta U}{\Delta t} = I^2 R \Rightarrow \Delta U = I^2 R \Delta t = (8.00)^2 (1.50)(3600) = \underline{\underline{346\text{kJ}}}$ .

(c) Using the definition of current density with the drift velocity equation,

$j = \frac{I}{A} = nev = ne \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = \frac{I \Delta t}{neA} = \frac{(8.00)(3600)}{(8.45 \times 10^{28})(1.60 \times 10^{-19})(0.300 \times 10^{-4})} = \underline{\underline{71.0\text{mm}}}$ .

4. For the circuit shown find (a) the equivalent resistance, (b) the current supplied by the battery, (c) the potential difference across each resistor and (d) the current through each resistor.  $V=18.0\text{V}$ ,  $R_1=300\Omega$ ,  $R_2=900\Omega$  and  $R_3=1800\Omega$ .



V(V)	I(mA)	R( $\Omega$ )
6.00	20.0	300
12.0	13.3	900
12.0	6.67	1800
battery 18.0	battery 20.0	equivalent 900

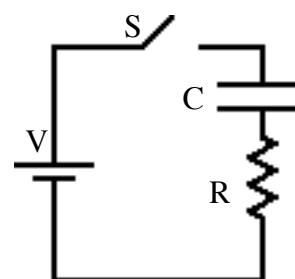
(a)  $R_2$  and  $R_3$  are in parallel so  $\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} \Rightarrow \frac{1}{R_p} = \frac{1}{900} + \frac{1}{1800} \Rightarrow R_p = 600\Omega$ .

$R_1$  is in series with  $R_p$  so  $R_{eq} = R_1 + R_p = 600\Omega + 300\Omega = 900\Omega$ .

(b) The current from the battery can be found using Ohm's Rule,  $V = iR \Rightarrow i = \frac{V}{R_{eq}} = \frac{18}{900} = 20.0\text{mA}$ .

(c)&(d) Since  $R_1$  is in series with the battery, the current through it is the same as the battery. The voltage on  $R_1$  can be found from Ohm's Rule. Using the loop theorem, the voltage on  $R_2$  and  $R_3$  must be equal and it must be the battery voltage less the voltage drop on  $R_1$ . From Ohm's Rule the current on  $R_2$  and  $R_3$  can be found.

5. A  $10.0\text{k}\Omega$  resistor is in series with a capacitor and a  $1.50\text{V}$  battery as shown at the right. 100s after the switch is closed the current in the circuit is  $20.0\mu\text{A}$ . Find the capacitance of the capacitor.



The charge on the capacitor varies with time according to,

$$q = CV(1 - e^{-t/RC}).$$

Using the definition of current,

$$I \equiv \frac{dq}{dt} = \frac{d}{dt} CV(1 - e^{-t/RC}) = CV \frac{1}{RC} e^{-t/RC} = \frac{V}{R} e^{-t/RC}.$$

Solving for the capacitance,

$$\frac{IR}{V} = e^{-t/RC} \Rightarrow \ln\left(\frac{IR}{V}\right) = -\frac{t}{RC} \Rightarrow C = \frac{t}{R \ln\left(\frac{V}{IR}\right)} = \frac{100}{(10000) \ln\left(\frac{1.50}{(20.0 \times 10^{-6})(10000)}\right)} = \underline{\underline{4.96\text{mF}}}.$$