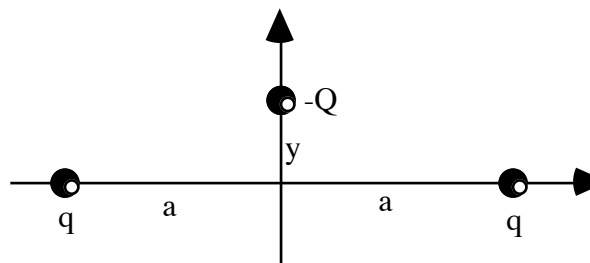


Name: _____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. The charge $-Q$ shown at the right is a distance y above the origin. Find (a) the electric potential it feels, (b) its potential energy, (c) its potential energy when it reaches the origin. (d) Will it be moving faster or slower when it reaches the origin? Explain in terms of the answers to parts (a), (b) and (c).



(a) Adding the potentials due to the point charges,

$$V = k \frac{q}{\sqrt{a^2 + y^2}} + k \frac{q}{\sqrt{a^2 + y^2}} = \frac{2kq}{\sqrt{a^2 + y^2}}$$

(b) Using the definition of electric potential,

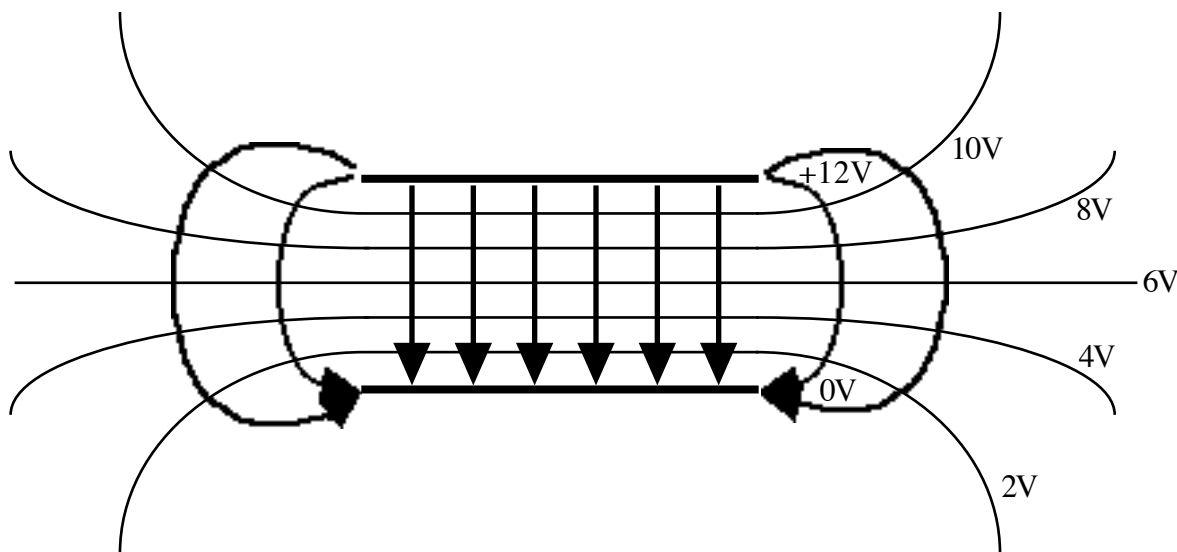
$$\Delta U \equiv q\Delta V = -Q \frac{2kq}{\sqrt{a^2 + y^2}} = -\frac{2kqQ}{\sqrt{a^2 + y^2}}.$$

(c) At the origin, $y = 0$ so,

$$\Delta U = -\frac{2kqQ}{a}$$

(d) Since the potential energy is now smaller (it is a bigger negative number) the kinetic energy must be larger by the Law of Conservation of Energy. Therefore, the charge must be moving faster.

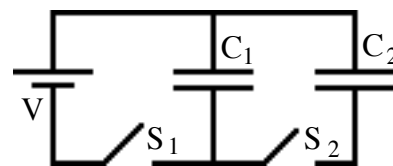
2. For the parallel plates shown below sketch, (a) the equipotential lines and (b) the field lines. Be sure to distinguish the equipotentials from the field lines. Explain (c) why equipotential lines must always be perpendicular to field lines and (d) why the field is stronger where the equipotential are crowded together.



(c) The relationship between field and potential is, $\Delta V = -\int \vec{E} \cdot d\vec{s}$. Moving in a direction $d\vec{s}$ perpendicular to \vec{E} causes the dot product to give zero for the change in potential. In other words, along the equipotentials the field must be perpendicular.

(d) For a given size ΔV , you can have a large E and a small displacement ds , or a small E and a large ds . So where the field E is large the distance ds between the equipotential is small and vice versa.

3. In the circuit at the right $V=1.50V$, $C_1=4000\mu F$ and $C_2=1400\mu F$. Initially the capacitors are uncharged. (a) Find the charge on the capacitors when switch S_1 is closed and switch S_2 is left open. (b) Now, S_1 is opened. Then S_2 is closed. Find the resulting charge on each of the capacitors.



(a) When S_1 is closed the voltage on C_1 is now $1.50V$.

Using the definition of capacitance, $C = \frac{Q}{V} \Rightarrow Q = C_1 V = (4000\mu F)(1.50V) = \underline{\underline{6000\mu C}}$.

(b) By the loop theorem, both capacitors must have the same voltage $V_1 = V_2$.

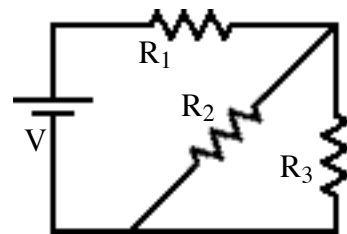
Using the definition of capacitance, $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$.

The total charge must be conserved so, $Q = Q_1 + Q_2 \Rightarrow Q_2 = Q - Q_1$.

Substituting into the equation from the loop theorem, $\frac{Q_1}{C_1} = \frac{Q - Q_1}{C_2} \Rightarrow Q_1 = \frac{Q}{\frac{C_2}{C_1} + 1} = \frac{6000}{\frac{1400}{4000} + 1} = \underline{\underline{4440\mu C}}$.

Using the charge conservation equation, $Q_2 = 6000 - 4440 = \underline{\underline{1560\mu C}}$.

4. In the circuit shown, the current through R_3 is measured to be 4.00mA. Find (a)the voltage across each resistor in the circuit, (b)the current through the other two resistors, (c)the current supplied by the battery, and (d)the voltage of the battery. $R_1=1.00\text{k}\Omega$, $R_2=3.00\text{k}\Omega$ and $R_3=6.00\text{k}\Omega$.



V(V)	I(mA)	R(k Ω)
12.0	12.0	1.00
24.0	8.00	3.00
24.0	4.00	6.00
battery 36.0	battery 12.0	X

The voltage across R_3 can be found using Ohm's Rule,

$$V = IR = (4)(6) = 24\text{V} .$$

Since R_2 and R_3 are in parallel the loop theorem requires that they have the same voltage drop. Now Ohm's Rule can be used to find the current through R_2 , $I = \frac{V}{R} = \frac{24}{3} = 8.00\text{mA} .$

By the junction theorem, the current through R_1 must equal the sum of the currents through R_2 and R_3 .

$$I_1 = I_2 + I_3 = 8 + 4 = 12\text{mA} .$$

The voltage across R_1 can be found from Ohm's Rule,

$$V = IR = (12)(1) = 12\text{V} .$$

The battery is in series with R_1 so it must have the same current.

The voltage of the battery must equal the voltage across R_1 plus either the voltage across R_2 or R_3 by the loop theorem. $V = V_1 + V_3 = 12 + 24 = 36\text{V} .$

5. A $20.0\mu\text{F}$ capacitor charged up to 9.00V, is shorted out with a piece of copper wire. The charge on the capacitor is 99% gone after 3.00ms. Find the resistance of the piece of wire.

This is the discharging of an RC circuit $q = CV_0 e^{-t/RC} .$

Using the definition of capacitance, $q = q_0 e^{-t/RC}$

$$\text{Solving for the resistance, } R = -\frac{t}{C \ln\left(\frac{q}{q_0}\right)} = -\frac{3.00 \times 10^{-3}}{20.0 \times 10^{-6} \ln\left(\frac{0.01q_0}{q_0}\right)} \Rightarrow \boxed{R = 32.6\Omega}$$