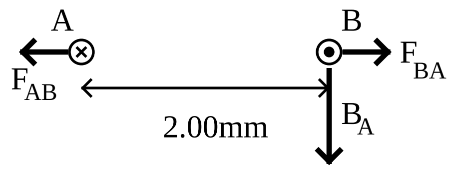
Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. The sketch at the right shows two wires that are perpendicular to the page. Wire A carries a current of 140mA into the page and wire B carries a current of 210mA out of the page. The wires are 2.00mm apart, 12.5μm in diameter and 2.60m long. Find (a)the magnetic field felt by wire B, (b)the force on wire B, (c)the force on wire A and (d)indicate the directions of the quantities in the sketch.



(a)The field felt by wire B is the field due to the long straight wire A,

$$B_A = \frac{\mu_o I_A}{2\pi r} = (2.00\text{x}10^{-7})\frac{140}{2.00} \qquad \boxed{B_A = 14.0\mu T}.$$

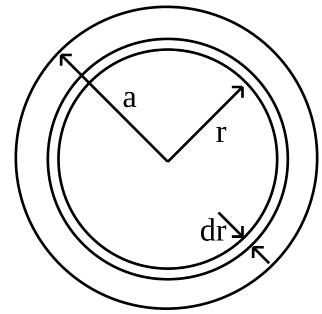
(b)The force on wire B is caused by the field of wire A. Using the definition of magnetic field,

$\vec{F}_{BA} = I_B \vec{\ell} \times \vec{B}_A$ since the field is perpendicular to the current flow,

$F_{BA} = I_B \ell B_A = (210\text{m})(2.60)(14.0\mu) \qquad \boxed{F_{BA} = 7.64\mu N}.$

(c)By the Third Law, the force on A must be equal and opposite to the force on B, $\boxed{F_{AB} = 7.64\mu N}$.

2. A circular wire of radius a carries a non-uniform current distribution. The current density is $j = j_o \frac{r}{a}$ where r is the distance from the center of the wire. Find (a)the total current in the wire and (b)the magnetic field inside the wire as a function of r.



(a)The definition of current density is $j \equiv \frac{I}{A}$ $\quad I = \int j dA$.
The current density here is constant over small rings of thickness dr. The area of these rings is given by, $dA = 2\pi r dr$. The total current through the wire is then,

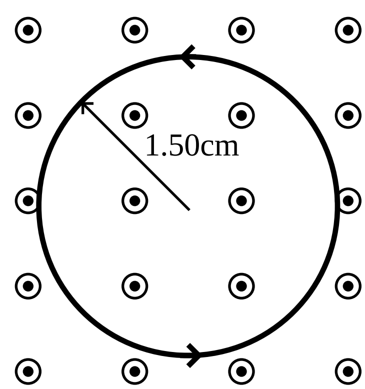
$$I = \int j dA = \int_0^a j_o \frac{r}{a} 2\pi r dr = \frac{2\pi j_o}{a}\int_0^a r^2 dr = \frac{2\pi j_o}{a}\frac{a^3}{3} \qquad \boxed{I = \frac{2\pi j_o a^2}{3}}.$$

(b)Applying Ampere's Law to the circular path of radius r, $\oint \vec{B} \cdot d\vec{s} = \mu_o i_{encl}$ $\quad B 2\pi r = \mu_o i_{encl}$ where we have used the fact that B must be constant along the path and it must point along the path by the circular symmetry.

The enclosed current is, $I = \int j dA = \int_0^r j_o \frac{r}{a} 2\pi r dr = \frac{2\pi j_o}{a}\int_0^a r^2 dr = \frac{2\pi j_o r^3}{3a}$.

The field is then, $B 2\pi r = \mu_o \frac{2\pi j_o r^3}{3a}$ $\qquad \boxed{B = \mu_o \frac{j_o r^2}{3a}}$.

3. The magnetic field through the circular loop of radius 1.50cm drops with time according to the equation $B = B_o(1 - \frac{t}{T})$ where B_o=200μT and T=100s. Find the induced voltage in the loop at t=50.0s and show the direction of the induced current.



Starting with Faraday's Law, $\oint \vec{E} \cdot d\vec{s} = -N\frac{d\Phi_B}{dt}$ $\qquad V = -\frac{d\Phi_B}{dt}$

then using the definition of flux, $V = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$.

Over the plane of the loop the field is constant and points along the area vector so,

$$V = -\frac{d}{dt} BA = -\frac{d}{dt} B_o A \left(1 - \frac{t}{T}\right)$$

Doing the derivative and plugging in the values,

$$V = \frac{B_o A}{T} = \frac{B_o \pi r^2}{T} = \frac{(200\mu)\pi(0.0150)^2}{100} \qquad \boxed{V = 1.41 \times 10^{-9} V}.$$

The direction of the current flow can be found from Lenz's Rule. Since the field out of the page is decreasing the current will be induced to create field out of the page as shown.

4. A 1.60V battery is connected to a coil of wire. After 10.0ms the current is 1.20mA. After 10.0s the current reaches a constant value of 4.80mA. Find the resistance and inductance of the coil.

After a long time, the voltage across the coil is just due to the resistance of the coil, so using Ohm's Rule,

$$V = IR \qquad R = \frac{V}{I} = \frac{1.60}{4.80m} \qquad \boxed{R = 333\Omega}.$$

For short times the equation for the "charging" of an inductor is relevant,

$$I = I_o\left(1 - e^{-\frac{R}{L}t}\right) \qquad L = -\frac{Rt}{\ln\left(1 - \frac{I}{I_o}\right)} = -\frac{(333)(0.0100)}{\ln\left(1 - \frac{1.20}{4.80}\right)} \qquad \boxed{L = 11.6H}.$$

5. In the lab you tested the expression for the field magnetic field due to a long straight wire by sending a current through the wire and detecting the magnetic field with a probe. The probe was made of a small coil of wire wrapped around an iron cylinder. The probe produced a sinusoidal voltage that you measured with the oscilloscope. Explain how the probe works. Be sure to state the names of the important principles for full credit.

The current through the straight wire is an alternating current. This alternating current creates an alternating magnetic field through the coil of the probe. According to Faraday's Law, the alternating field through the coil is a changing flux that induces an oscillating voltage in the coil.