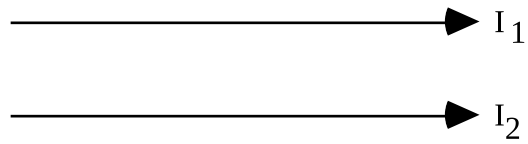
Physics 4B THIRD EXAM Chapters 23 - 32 Spring 1995

Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. Two long straight parallel wires carry currents I_1=5.00A and I_2=7.00A. They are 12.0cm apart. Find (a)the magnetic field felt by I_1, (b)the magnetic field felt by I_2, and (c)the force per unit length that I_1 exerts on I_2. Be sure to state the directions as well as the magnitudes.



(a)The field felt by wire 1 is produced by the long straight wire 2:

$$\boxed{B_2 = \frac{\mu_o I_2}{2\pi r} = (2.00x10^{-7})\frac{7.00}{0.120} = 11.7\mu T}$$

. This field is out of the paper by the right hand rule.

(b)The field felt by wire 2 is produced by the long straight wire 1:

$$\boxed{B_1 = \frac{\mu_o I_1}{2\pi r} = (2.00x10^{-7})\frac{5.00}{0.120} = 8.33\mu T}$$

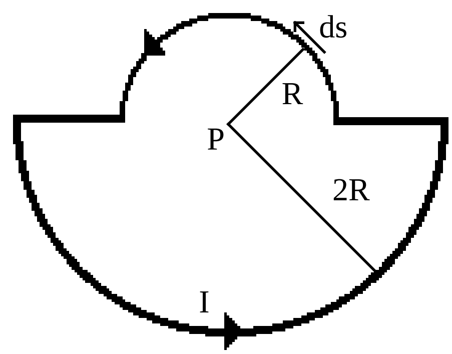
. This field is into the paper by the right hand rule.

(b)The force felt by wire 2 is produced by the field of wire 1 acting on the current in wire 2. Using the definition of magnetic field:

$$\vec{F} \equiv I\vec{\ell} \times \vec{B} \qquad F = I_2 \ell B_1 \quad \boxed{\frac{F}{\ell} = I_2 B_1 = (7.00)(8.33\mu) = 58.3\mu N}$$

. This force is upward toward wire 1 by the right hand rule.

2. Find the expression for the magnitude and indicate the direction of the magnetic field at the point P which is at the common center of the semi-circular arcs shown in the sketch.



Use the Biot-Savart Rule $\vec{B} = \frac{\mu_o}{4\pi}\int \frac{Id\vec{s} \times \hat{r}}{r^2}$.

Along the top arc, ds is always perpendicular to R so, $B_t = \frac{\mu_o}{4\pi}\int \frac{Ids}{R^2}$.

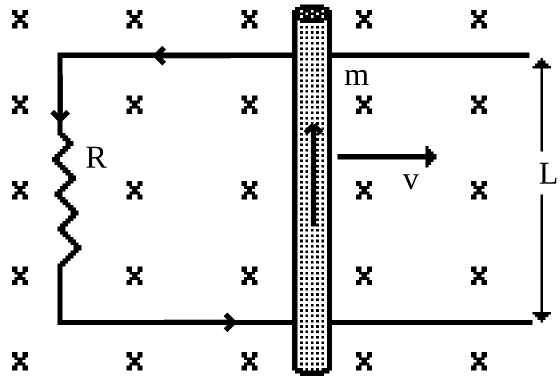
Since I and R are constant, $B_t = \frac{\mu_o I}{4\pi R^2}\int ds = \frac{\mu_o I}{4\pi R^2}\pi R = \frac{\mu_o I}{4R}$.

Along the straight sections ds is always parallel to R so they create no field at P.

The contribution of the bottom arc is the same as the top except the radius is twice as big, $B_b = \frac{\mu_o I}{8R}$.

The total field is $\boxed{B = B_t + B_b = \frac{\mu_o I}{4R} + \frac{\mu_o I}{8R} = \frac{3\mu_o I}{8R}}$. This field is out of the page by the right hand rule.

3. A bar of mass, m, and length, L, moves on two frictionless parallel rails of total resistance, R, in the presence of a uniform magnetic field, B, directed into the paper. The bar is given an initial velocity, v_o, to the right and is released. Sometime later when the bar is moving at a speed, v, find (a)the direction and magnitude of the current induced in the bar, (b)the direction and magnitude of the force on the bar, (c)the acceleration of the bar, and (d)the speed of the bar as a function of time.



(a)Using Faraday's Law the induced voltage is,

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad V = \frac{d\Phi_B}{dt}.$$

Using Ohm's Rule and the definition of flux, $IR = \frac{d}{dt}\int \vec{B}\cdot d\vec{A}$.

Since the field is parallel to the area vector and constant, $IR = \frac{d}{dt}BA$.

The field doesn't change with time and the rate at which area is swept out is $\frac{dA}{dt} = vL$. Solving for the current,

$\boxed{I = \frac{BvL}{R}}$. The direction is given by Lenz' Rule to be counterclockwise as shown.

(b)The force on the bar can be found from the definition of magnetic field $\vec{F} \equiv I\vec{\ell}\times\vec{B}$.

Since the current is perpendicular to the field, $\boxed{F = -ILB = -\frac{B^2vL^2}{R}}$.

The direction is to the left opposite the velocity by the right hand rule.

(c)Applying the Second Law, $\Sigma F = ma$ and solving for acceleration $\boxed{a = \frac{F}{m} = -\frac{B^2vL^2}{mR}}$.

(d)Using the definition of acceleration, $a \equiv \frac{dv}{dt} \qquad dv = a\,dt \qquad dv = -\frac{B^2vL^2}{mR}dt \qquad \frac{dv}{v} = -\frac{B^2L^2}{mR}dt$.

Integrating, $\int_{v_o}^{v}\frac{dv}{v} = -\int_0^t \frac{B^2L^2}{mR}dt \qquad \ln\frac{v}{v_o} = -\frac{B^2L^2}{mR}t$ and solving $\boxed{v = v_o \exp\left(-\frac{B^2L^2}{mR}t\right)}$

4. Find the induced voltage across a 10.0mH inductor after 5.00s when the current through the inductor (a)is a constant 300mA, (b)increases linearly at a rate of 4.00mA/s, and (c)decreases exponentially according to $i = (1.00A)e^{-\frac{t}{10s}}$.

Use the definition of inductance is $\mathcal{E} \equiv -L\frac{dI}{dt}$.

(a)In this case the current is constant so $\frac{dI}{dt} = 0$ and therefor there will be no induced voltage $\boxed{\mathcal{E} = 0}$.

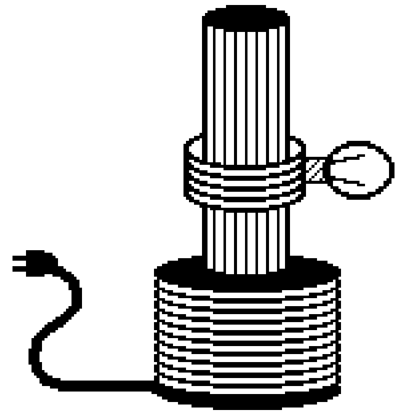
(b)Here $\frac{dI}{dt} = 4.00mA/s$ and remains constant so $\boxed{\mathcal{E} = -(0.010)(0.004) = -40.0\mu V}$.

(c)Now the current changes with time $\frac{dI}{dt} = \frac{d}{dt}(1.00A)e^{-\frac{t}{10s}} = -(0.100A/s)e^{-\frac{t}{10s}}$

and so does it's derivative.

At t=5.00s, $\frac{dI}{dt} = -(0.100A/s)e^{-\frac{1}{2}} = -60.7mA/s$ so $\boxed{\mathcal{E} = -(0.010)(-0.0607) = 607\mu V}$.

5. In the lab, you studied the behavior of a coil of wire wrapped around an iron core. The coil was connected to the wall current (110V at 60Hz). When the current was turned on in the coil, a small light bulb with its own smaller coil, also went on. Explain why the bulb lights.



This is an example of Faraday's Law. The current in the coil creates a magnetic field in the iron core. Since the current is changing this magnetic field is changing. The changing magnetic field through the smaller coil induces a voltage according to Faraday's Law. This voltage creates the current that lights the bulb.