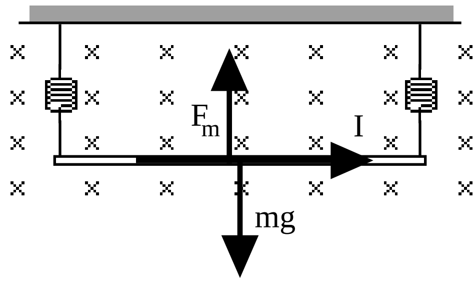
Name:____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 20 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A 62.0cm long wire has a mass of 130g and is hung horizontally from two conducting springs. The wire is perpendicular to an 0.880T magnetic field directed into the page as shown. Find the magnitude and direction of the current in the wire if the springs are unstretched.



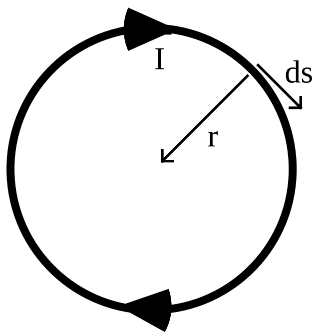
According to the right hand rule, the current must be moving toward the right in order to feel an upward force.

Starting with Newton's Second Law,

$\Sigma F = ma \qquad F_m - mg = 0 \qquad F_m = mg$.

Using the definition of magnetic field $I\ell B = mg \qquad I = \dfrac{mg}{\ell B} = \dfrac{(0.130)(9.80)}{(0.620)(0.880)}$ $\boxed{I = 2.34A}$.

2. Find the magnitude and direction of the magnetic field at the center of the coil shown at the right. The coil has 100 turns, a radius of 5.00cm and carries 1.00A of current.



Starting with the Biot-Savart Rule, $\vec{B} = \dfrac{\mu_o}{4\pi}\int \dfrac{I d\vec{s} \times \hat{r}}{r^2}$.

In this case, $d\vec{s} \times \hat{r}$ point into the paper and has a magnitude of ds.

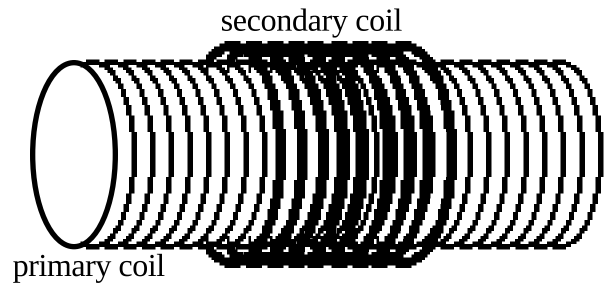
Since the radius and current are constant, $B = \dfrac{\mu_o I}{4\pi r^2}\int ds$.

The integral gives the circumference times the number of turns,

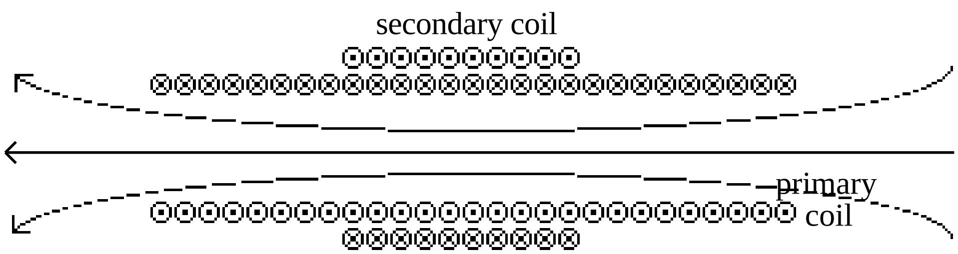
$B = \dfrac{\mu_o I}{4\pi r^2} 2\pi r N = \dfrac{\mu_o NI}{2r}$.

Putting in the numbers, $\boxed{B = \dfrac{(4\pi x 10^{-7})(100)(1.00)}{2(0.0500)} = 1.26mT}$

3. At the right is a drawing of the concentric coils that you studied in the lab. Current is provided to the primary coil by a battery. When the magnetic field of the primary coil has the right properties, currents appear in the secondary coil. The system is shown below in cross section for three cases. For each case sketch the magnetic field of the primary coil and indicate the direction of the current in the secondary coil. State the concepts or principles that you use to determine your answer and explain your reasoning.

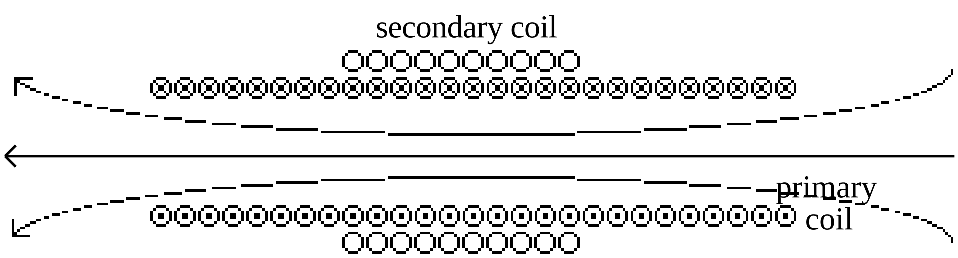


Case 1: The battery was recently connected. The current in the primary is in the direction shown and growing.



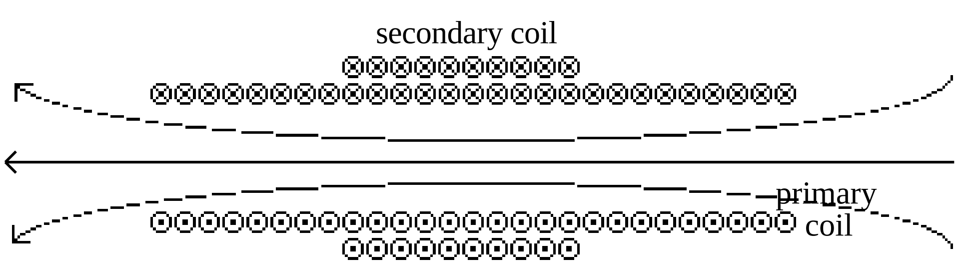
Since the field to the left is increasing, according to Faraday's Law and Lenz's Rule, the secondary coil will produce a field that fights the increase. Therefor, the field due to the secondary must be opposite to the field of the primary.

Case 2: The battery was connected long ago. The current in the primary is in the direction shown and constant.



Since the field is constant, according to Faraday's Law, the secondary coil will produce no field.

Case 3: The battery was recently disconnected. The current in the primary is in the direction shown and dropping.



Since the field to the left is decreasing, according to Faraday's Law and Lenz's Rule, the secondary coil will produce a field that fights the decrease. Therefor, the field due to the secondary must be in the same direction as the field of the primary.

4. Find the induced voltage across a 10.0mH inductor after 5.00s when the current through the inductor (a)is a constant 300mA, (b)increases linearly at a rate of 4.00mA/s, and (c)decreases exponentially according to $I = I_o e^{-\frac{t}{T}}$ where I_o=1.00A and T=10.0s.

All three questions require the use of the definition of inductance, $\mathcal{E} \equiv -L\frac{dI}{dt}$

(a)The current is constant, so $\frac{dI}{dt} = 0$. Therefor, there will be no induced voltage, $\boxed{\mathcal{E} = 0}$.

(b)In this case, $\frac{dI}{dt} = 4.00\text{mA/s}$. Therefor, the induced voltage is $\mathcal{E} = -(10.0\text{m})(4.00\text{m})$ $\boxed{\mathcal{E} = -40.0\mu\text{V}}$.

(c)Now the current changes exponentially with time so, $\frac{dI}{dt} = \frac{d}{dt} I_o e^{-\frac{t}{T}} = -\frac{I_o}{T} e^{-\frac{t}{T}}$.

The induced voltage will be, $\mathcal{E} = -L\frac{dI}{dt} = \frac{LI_o}{T} e^{-\frac{t}{T}}$.

At t=5.00s, $\mathcal{E} = \frac{(10.0\text{m})(1.00)}{10.0} e^{-\frac{5.00}{10.0}}$ $\boxed{\mathcal{E} = 607\mu\text{V}}$.

5. Magnetic resonance imaging is done instead of x-rays for medical diagnosis. It works by measuring the energy released by protons in the body when their magnetic moment flips from being anti-parallel to parallel with the applied magnetic field. The magnetic dipole moment of a proton is $1.41\text{x}10^{-26}$J/T. Find the strength of the required field if the minimal detectable energy is one millionth of an electron-volt.

The potential energy of a dipole is, $U = -\vec{\mu} \cdot \vec{B}$.
When it is anti-parallel, $U_i = \mu B$ and when it is parallel, $U_f = -\mu B$.
The change in the potential energy is then, $\Delta U = U_f - U_i = -\mu B - (\mu B) = -2\mu B$.
By the Law of Conservation of Energy, the energy released will be, $E = 2\mu B$.
Solving for the field, $B = \frac{E}{2\mu} = \frac{(10^{-6}\text{eV})(1.60\text{x}10^{-19}\ \text{J/eV})}{2(1.41\text{x}10^{-26}\ \text{J/T})}$ $\boxed{B = 5.67\text{T}}$.