Physics 4B FINAL EXAM Chapters 23 - 34 Fall 1995

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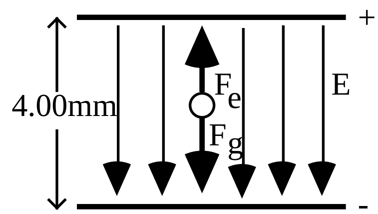
Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You <u>must</u> show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. A small plastic sphere of mass $5.00\text{x}10^{-16}$kg is held motionless between two charged parallel plates 4.00mm apart. Assume that the plastic sphere has two excess electrons on it. Find (a)the electric force on the sphere, (b)the electric field between the plates and (c)the potential difference between the plates. (d)In the diagram at the right, show the polarity of the plates, sketch the electric field and show all the forces that act on the sphere.



(a)According to Newton's Second Law, $\Sigma F = ma \qquad F_e - F_g = 0 \qquad F_e = F_g$.

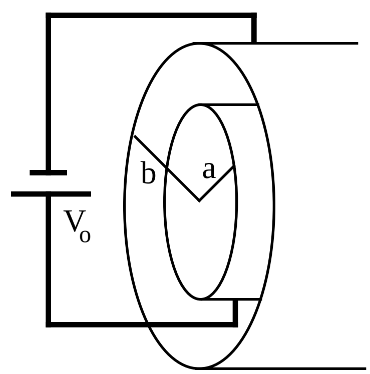
Putting in the numbers, $F_e = mg = (5.00\text{x}10^{-16})(9.80) = \underline{\underline{4.90\text{x}10^{-15}\text{N}}}$.

(b)Using the definition of electric field, $\boxed{E \equiv \frac{F}{q} = \frac{4.90\text{x}10^{-15}}{2(1.60\text{x}10^{-19})} = \underline{\underline{1.53\text{x}10^{4}\,\frac{\text{V}}{\text{m}}}}}$.

(c)The electric potential is $\Delta V = -\int \vec{E}\cdot d\vec{s}$.

Since the field between the plates is constant, $V = Ed = (1.53\text{x}10^{4})(4.00\text{x}10^{-3}) = \underline{\underline{61.3\text{V}}}$.

2. Two long concentric cylinders of radii a and b have a potential difference V_o established between them. Find the electric field between the cylinders as a function of the distance from the center. (Hint: You may want to start by assuming a linear charge density λ exists on the cylinders, but for full credit you must express λ in terms of the potential difference).



Applying Gauss's Law to an imaginary cylinder with a radius r between a and b. Since the cylinders are long, symmetry requires the field to be constant over this Gaussian cylinder as it must point radially outward. Therefor,

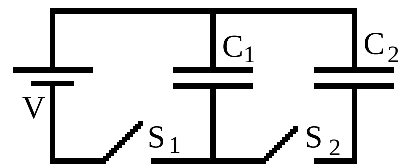
$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_o} \qquad E2\pi r\ell = \frac{\lambda\ell}{\varepsilon_o} \qquad E = \frac{\lambda}{2\pi\varepsilon_o r}$$

The electric potential is $\Delta V = -\int \vec{E}\cdot d\vec{s}$. Integrating from the inner to the outer cylinder,

$$0 - V_o = -\int_a^b \frac{\lambda}{2\pi\varepsilon_o r}dr = \frac{\lambda}{2\pi\varepsilon_o}\ln\left(\frac{b}{a}\right) \qquad \lambda = \frac{2\pi\varepsilon_o V_o}{\ln\left(\frac{b}{a}\right)}.$$

$$\text{Finally, } E = \frac{\lambda}{2\pi\varepsilon_o r} = \frac{2\pi\varepsilon_o V_o}{2\pi\varepsilon_o r\ln\left(\frac{b}{a}\right)} \qquad \underline{\underline{\frac{V_o}{r\ln\left(\frac{b}{a}\right)}}}.$$

3. In the circuit at the right $V=1.50V$, $C_1=4000\mu F$ and $C_2=1400\mu F$. Initially the capacitors are uncharged. (a)Find the charge on the capacitors when switch S_1 is closed and switch S_2 is open. (b)Now, S_1 is opened. Tnen S_2 is closed. Find the resulting charge on each of the capacitors.



(a)When S_1 is closed the voltage on C_1 is now 1.50V.

Using the definition of capacitance, $C \equiv \frac{Q}{V}$ $\quad Q = C_1 V = (4000\mu F)(1.50V) = \underline{\underline{6000\mu C}}$.

The total charge must be conserved so, $Q = Q_1 + Q_2 \qquad Q_2 = Q - Q_1$.

Substituting into the equation for V, $\frac{Q_1}{C_1} = \frac{Q - Q_1}{C_2} \qquad Q_1 = \frac{Q}{\frac{C_2}{C_1} + 1} = \frac{6000}{\frac{1400}{4000} + 1} = \underline{\underline{4440\mu C}}$.

Using the charge conservation equation, $Q_2 = 6000 - 4440 = \underline{\underline{1560\mu C}}$.

4. An RC circuit contains a 10.0±0.1kΩ resistor. The time for the voltage on the resistor to drop from 1.50V down to 0.750V is 15.2±0.2s. Find (a)the capacitance of the circuit and (b)the uncertainty in the capacitance. Assume the voltages are exact. The manufacturer claims the capacitance is 2.00mF within 10%. (c)Which number should you use for the capacitance, the experimental or the manufacturer's? Explain.

(a)For a discharging capacitor $q = CV_o e^{-t/RC}$.

Using the definition of capacitance, $V = V_o e^{-t/RC}$

Solving for the capacitance, $C = \frac{t}{R\ln\left(\frac{V_o}{V}\right)} = \frac{15.2}{10000\ln(2)} = \underline{\underline{2.19\text{mF}}}$

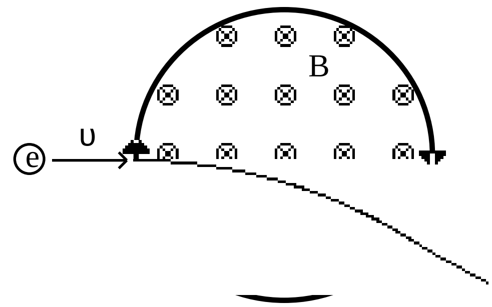
(b)The uncertainty equation is from the multiplication rule is,

$\Delta C = C\sqrt{\left(\frac{\Delta R}{R}\right)^2 + \left(\frac{\Delta t}{t}\right)^2} \qquad \Delta C = (2.19)\sqrt{\left(\frac{0.1}{10.0}\right)^2 + \left(\frac{0.2}{15.2}\right)^2} = 0.04\text{mF}$.

Using proper significant figures, $\boxed{C = 2.19 \pm 0.04\text{mF}}$.

(c)While both numbers are in agreement, the experimental value is the one to use because it has a smaller uncertainty.

5. An electron is accelerated by a potential difference of 3000V into a region of nearly constant magnetic field created by the current in a pair of Helmholtz coils of radius 8.00cm as shown at the right. As a result the electrons bend along a path with a radius of curvature of 26.0cm. Find (a)the speed of the electron as it enters the magnetic field, (b)the magnitude of the magnetic field, and (c)the speed of the electron as it leaves the magnetic field. (d)In the diagram, sketch the path of the electrons and direction of the current in the coils. For simplicity, assume the magnetic field is constant and into the page inside the coils and zero outside.



(a)As the electrons are accelerated they lose 3000eV of potential energy.
By the Law of Conservation of Energy they gain 3000eV of kinetic energy,

$$\mathcal{E}_k = \tfrac{1}{2}mv^2 \qquad v = \sqrt{\frac{2\mathcal{E}_k}{mc^2}}\,c = \sqrt{\frac{2(3000)}{5.11\text{x}10^5}}(3.00\text{x}10^8) = \underline{\underline{3.25\text{x}10^7\ \tfrac{\text{m}}{\text{s}}}}.$$

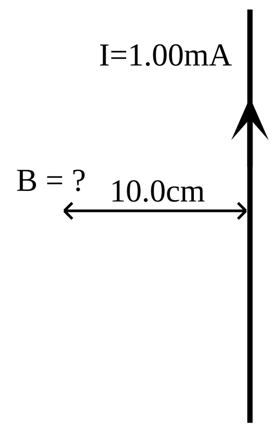
(b)Using Newton's Second Law with the force on a charged particle in a magnetic field and the centripetal acceleration of a circular orbit,

$$\Sigma F = ma \qquad qvB = m\frac{v^2}{r} \qquad B = \frac{mv}{qr} = \frac{(9.11\text{x}10^{-31})(3.25\text{x}10^7)}{(1.60\text{x}10^{-19}\,\text{C})(0.26)} = \underline{\underline{712\,\mu\text{T}}}.$$

(c)Since the magnetic force is always perpendicular to the motion of the electron, there is no work done on it. Therefor its kinetic energy and speed are unchanged. $v = \underline{\underline{3.25\text{x}10^7\ \tfrac{\text{m}}{\text{s}}}}$.

(d)The electrons bends along a circular arc inside the field, then heads off in a straight line after leaving the field. The current in the coils must be dlockwise by the right hand rule.

6. (a)Find the magnetic field 10.0cm away from the center of a straight wire carrying a current of 1.00mA assuming the wire is infinitely long. (b)Since the wire you used in lab was only about 50.0cm long, explain how you would find the field due to this wire of finite length and indicate whether the result should be larger or smaller than the value from part a. Note that you are not asked to actually find the field, just explain how you would.

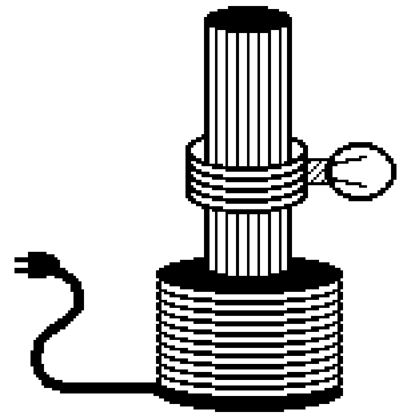


(a)The field due to a long straight wire is, $B = \frac{\mu_o I}{2\pi r} = (2.00\text{x}10^{-7})\frac{0.001}{0.500} = \underline{\underline{4.00\text{x}10^{-10}\,\text{T}}}$

(b)The field can be found using the Biot-Savart Rule, $\vec{B} = \frac{\mu_o}{4\pi}\int\frac{Id\vec{s}\times\hat{r}}{r^2}$.

It will be less than the field due to the long straight wire because there are fewer current segments to contribute to the field.

7. In the lab, you studied the behavior of a coil of wire wrapped around an iron core. The coil was connected to the wall current (110V at 60Hz). When the current was turned on in the coil, a small light bulb with its own smaller coil, also went on. Explain why the bulb lights.



This is an example of Faraday's Law. The current in the coil creates a magnetic field in the iron core. Since the current is changing this magnetic field is changing. The changing magnetic field through the smaller coil induces a voltage according to Faraday's Law. This voltage creates the current that lights the bulb.

8. A 1000 turn coil with a radius of 6.00cm spins with a period of 50.0ms. The axis of the coil is horizontal and points eastward, while the earth's magnetic field of 52.0μT points northward at 62˚ below horizontal. Find the peak voltage induced in the coil.

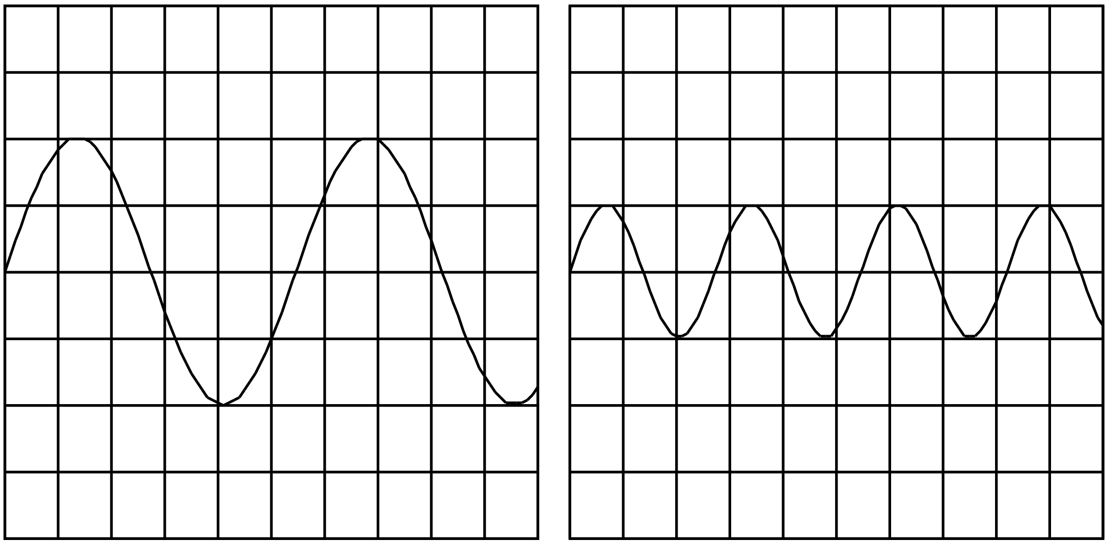
Since the coil is pointed eastward, the axis of the coil is always perpendicular to the magnetic field. The field makes an angle θ with the normal to the plane of the loop. As the coil spins, this angle changes at a constant rate. Starting with the definition of flux and using the fact that the field is constant, $\Phi_B \equiv \int \vec{B}\cdot d\vec{A} = BA\cos\theta$

Using Faraday's Law $V = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}(BA\cos\theta) = NBA\sin\theta\frac{d\theta}{dt}$.

The voltage peaks when $\sin\theta = 1$. Putting in the numbers,

$$V_o = NBA\frac{d\theta}{dt} = NB\pi r^2\frac{2\pi}{T} = \frac{2NB\pi^2 r^2}{T} = \frac{2(1000)(52.0x10^{-6})\pi^2(0.060)^2}{0.050} = \underline{\underline{73.9mV}}$$

9. An RLC series circuit contains a 680mH inductor, a 1.00nF capacitor, and a 2.50kΩ resistor. An oscilloscope set on 1volt/division is placed across the resistor at resonance. It's trace is shown in the graph at the left. (a)Find the scale on the horizontal axis (Hint: It should be in multiples of ten.) (b)Sketch the oscilloscope trace if the frequency is increased by 1000Hz. You don't need to get the amplitude or period exactly right for full credit.



(a)The resonance frequency is $\omega_o = \frac{1}{\sqrt{LC}}$.

The period should be $\frac{2\pi}{T} = \frac{1}{\sqrt{LC}}$ $\quad T = 2\pi\sqrt{LC} = 2\pi\sqrt{(0.680)(1.00\text{x}10^{-9})} = 164\mu s$.

Since one period is about 5.5 divisions, $\frac{164\mu s}{5.5\text{div}} \approx 30\mu s/\text{ div}$.

(b)As the frequency is moved above the resonance, the resulting current is reduced and therefor the voltage on the resistor drops. The amplitude of the voltage trace will be smaller as the frequency is higher.

10. The lights in the laboratory are 40.0W fluorescent bulbs which are cylindrical tubes about 2m long. Assume that all the light is emitted radially from these tubes at an average wavelength of 550nm. Estimate (a)the intensity of the electromagnetic waves produced by a single bulb 1.00m away from it, (b)the magnitude of the electric field in these electromagnetic waves, and (c)the speed of these electromagnetic waves.

(a)The intensity is the power per unit area. The area in this case is a cylinder of 1.00m radius.

$$S_{av} = \frac{P}{A} = \frac{P}{2\pi r\ell} = \frac{40.0}{2\pi(1.00)(2)} = \underline{\underline{3.18\frac{W}{m^2}}}$$

(b)The average intensity is the Poynting Vector, $S_{av} = \frac{E_m^2}{2\mu_o c}$ $\quad E_m = \sqrt{2\mu_o c S_{av}}$.

Putting in the numbers, $E_m = \sqrt{2(4\pi\text{x}10^{-7})(3\text{x}10^8)(3.18)} = \underline{\underline{49.0\frac{V}{m}}}$.

(c)The speed of EM waves is always the speed of light $3.00\text{x}10^8$m/s.