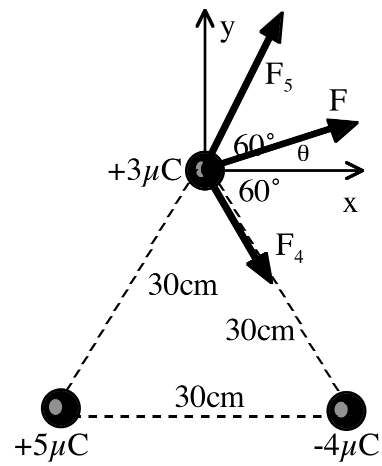
Name:____ Posting Code _____ _____ _____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. (a)Find the magnitude and direction of the electric force on the 3μC charge.
(b)Find the magnitude and direction of the electric field felt by the 3μC charge.



(a)The sizes of the forces can be found using Coulomb's Rule,

$$F_5 = (9.00\text{x}10^9)\frac{(3.00\text{x}10^{-6})(5.00\text{x}10^{-}6)}{(0.300)^2} = 1.50\text{N}$$

$$F_4 = (9.00\text{x}10^9)\frac{(3.00\text{x}10^{-6})(4.00\text{x}10^{-}6)}{(0.300)^2} = 1.20\text{N}$$

Adding the vector components,

$F_x = F_5\cos 60^\circ + F_4\cos 60^\circ = 1.35\text{N}$ and $F_y = F_5\sin 60^\circ - F_4\sin 60^\circ = 0.260\text{N}$

Find the magnitude using the Pythagorean Theorem,

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(1.35)^2 + (0.260)^2} = \underline{\underline{1.37\text{N}}}$$

Find the direction using the defintion of tangent,

$$\tan\theta = \frac{F_y}{F_x} \Rightarrow \theta = \arctan\frac{F_y}{F_x} = \arctan\frac{0.260}{1.35} = \underline{\underline{10.9^\circ}}$$

(b)Using the defintion of E-Field $\vec{E} \equiv \frac{\vec{F}}{q}$

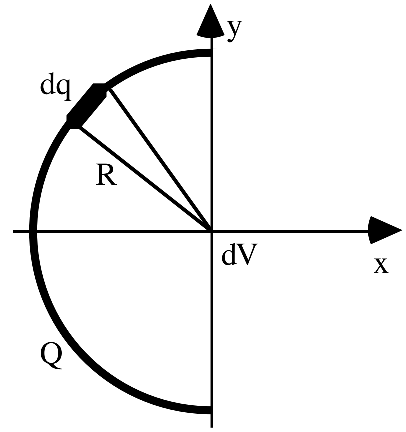
$$E = \frac{F}{q} = \frac{1.37}{3.00\text{x}10^{-6}} = \underline{\underline{4.57\text{x}10^5\text{N/C}}}$$

Since the charge feeling the field is positive, the direction of the field is the <u>same as the direction of the force</u>.

2. State Gauss's Law in words (Don't just "write the equation" with words and don't use the word "flux" unless you explain what it means.). Explain why it works. Discuss the conditions that must be met to use it to find the electric field due to a charge distribution.

Gauss's Law states that the total number of electric field lines coming out of any volume is proportional to the net amount of charge contained in the volume. This works because each unit of charge produces the same number of field lines. In order to use Gauss's Law to find the electric field, the charge distribution must have a high degree of symmetry.

3. Find the potential at the origin due to a semicircular arc of radius R and total charge Q.



The potential due to the point charge dq is:

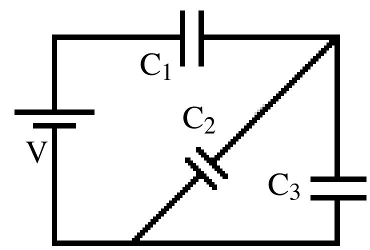
$$dV = k\frac{dq}{R}$$

There is no need to worry about components for potential because it isn't a vector. The total potential is,

$$V = \int k\frac{dq}{R} = k\frac{1}{R}\int dq$$

Finally, $\boxed{V = k\frac{Q}{R}}$

4. The charge on C_3 shown in the circuit at the right is 8.00μC. Find (a)the potential difference across each capacitor, (b)the charge on each capacitor and (c)the voltage of the battery (C_1=4.00μF, C_2=6.00μF and C_3=12.0μF).



Q(μC)	C(μF)	V(V)
12.0	4.00	3.00
4.00	6.00	0.667
8.00	12.00	0.667

$V_{battery} = 3.67V$

The voltage across C_3 can be found using the definition of capacitance,

$$V_3 = \frac{Q_3}{C_3} = \frac{8}{12} = 0.667V.$$

Since C_3 is in parallel with C_2, they both have the same potential difference. Now, the charge on C_2 can be found,

$$Q_2 = C_2V_2 = (6)(0.667) = 4.00\mu C.$$

Since C_1 is in series with the combination of C_2 and C_3, the charge on C_1 must equal the sum of the charges on C_2 and C_3.

$$Q_1 = Q_2 + Q_3 = 4 + 8 = 12.0\mu C$$

The voltage across C_1 is,

$$V_1 = \frac{Q_1}{C_1} = \frac{12}{4} = 3.00V$$

By the loop theorem, the voltage on the battery should be the sum of the voltage on C_1 and either C_2 or C_3.

$$V = V_1 + V_3 = 3 + 0.667 = 3.67V$$

5. A 100W light bulb uses a 120V-rms power source. Find (a)the rms current through the bulb and (b)the resistance of the filament.

(a)The electric power is $P = IV \Rightarrow I = \frac{P}{V} = \frac{100\,W}{120V} = \underline{\underline{833mA}}$.

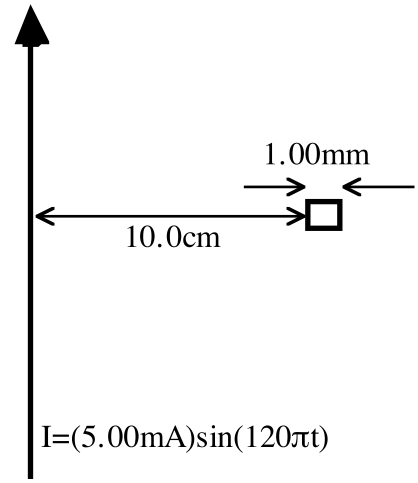
(b)Using Ohm's Rule, $V = IR \Rightarrow R = \frac{V}{I} = \frac{120V}{0.833} = \underline{\underline{144\Omega}}$.

6. At some location, the magnetic field of the Earth is 50.0μT vertically downward. A proton moving horizontally toward the west has a velocity of 6.20x10^6m/s. Find (a)the magnitude and direction of the magnetic force on the proton and (b)the radius of its circular path.

(a)The magnetic force on a moving charge is $\vec{F} = q\vec{v} \times \vec{B}$.
Since the velocity and field are perpendicular $F = qvB = (1.60x10^{-19})(6.20x10^{6})(50.0x10^{-6}) = \underline{\underline{4.96x10^{-17}N}}$.
The direction is determined by the right hand rule. The force is directed $\underline{\underline{\text{southward}}}$.

(b)Using Newton's Second Law and the centripetal acceleration,
$$\Sigma F = ma \Rightarrow F = m\frac{v^2}{r} \Rightarrow r = \sqrt{\frac{mv^2}{F}} = \sqrt{\frac{(1.67x10^{-27})(6.20x10^{6})^2}{4.96x10^{-17}}} = \underline{\underline{36.0m}}$$

7. A wire carries a peak current of 5.00mA that oscillates sinusoidally at 60.0Hz. A 1000 turn square coil lies in the same plane 10.0cm away. The sides of the coil are 1.00mm long. Find the peak voltage induced in the coil.



Using Faraday's Law, $V = -N\frac{d\Phi}{dt}$, and the definition of magnetic flux, $\Phi \equiv \int B \bullet dA = BA$ since the area of the coil is small the field through it is nearly constant. The field is due to a long straight wire so,

$$V = -N\frac{d}{dt}(BA) = -N\frac{d}{dt}\left(\frac{\mu_o I}{2\pi r}A\right) = -N\frac{\mu_o A}{2\pi r}\frac{dI}{dt} = -N\frac{\mu_o A}{2\pi r}2\pi f I_o \cos(2\pi f t).$$

The peak voltage occurs when the cosine is one.

$$V_o = N\frac{\mu_o A f I_o}{r} = (1000)\frac{(4\pi x10^{-7})(10^{-3})^2(60)(5x10^{-3})}{0.100} = \underline{\underline{3.77x10^{-9}\,V}}$$

8. A 1.00±0.05μH inductor is used in a radio tuner with a variable capacitor. Find (a)the capacitance required to have the tuner circuit oscillate at the same frequency as the radio station 93.9±0.1MHz and (b)the uncertainty in this capacitance.

(a)An LC circuit will oscillate at an angular frequency given by $\omega = \frac{1}{\sqrt{LC}} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$.

Solving for the capacitance $C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2(93.9x10^6)^2(1.00x10^{-6})} = \underline{\underline{2.87\,pF}}$

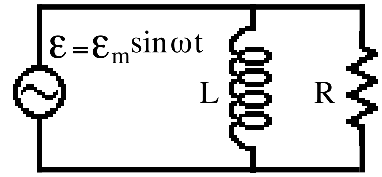
(b)Using the multiplication rule,

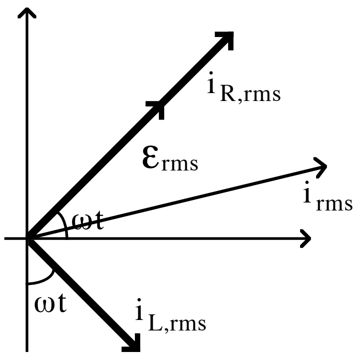
$$\frac{\Delta C}{C} = \sqrt{\left(2\frac{\Delta f}{f}\right)^2 + \left(\frac{\Delta L}{L}\right)^2} = \sqrt{\left(2\frac{0.1}{93.9}\right)^2 + \left(\frac{0.05}{1.00}\right)^2} = 0.0500$$

$\Delta C = (0.0500)C = (0.0500)(2.87) = 0.144\,pF$

Using proper significant figures, $\underline{\underline{C = 2.9 \pm .1 pF.}}$

9. A 350mH inductor is connected in parallel with a 50.0Ω resistor and a 60.0Hz - 15.0V rms power supply. (a)Draw the phasor diagram for this circuit. Find the rms voltage across (b)the power supply, (c)the inductor and (d)the resistor. Find the rms current (e)through the inductor, (f)through the resistor and (g)provided by power supply.





(a)

Note that since the circuit elements are in parallel, they all have the same voltage phasor.

(b)This is given to be $\boxed{\mathcal{E} = 15.0\,V}$.

(c)Since the inductor is in parallel with the power supply the loop theorem requires the voltages to be equal, $\boxed{V_{L,rms} = 15.0\,V}$.

(d)Since the resistor is also in parallel with the power supply the loop theorem requires, $\boxed{V_{R,rms} = 15.0\,V}$.

(e)Using the definition of impedance and the impedance of a inductor,

$$V_{L,rms} = i_{L,rms}\chi_C \Rightarrow i_{L,rms} = \frac{V_{L,rms}}{\chi_L} = \frac{V_{L,rms}}{2\pi f L}$$

Putting in the numbers, $\boxed{i_{L,rms} = \frac{15}{2\pi(60)(0.350)} = 114\,mA}$.

(f)Using the definition of impedance and the impedance of a resistor,

$$V_{R,rms} = i_{R,rms}\chi_R \Rightarrow i_{R,rms} = \frac{V_{R,rms}}{\chi_R} = \frac{V_{R,rms}}{R}$$

Putting in the numbers, $\boxed{i_{R,rms} = \frac{15}{50} = 300\,mA}$.

(g)Applying the Pythagorean Theorem to the phasor diagram,

$\boxed{i_{rms} = \sqrt{i_{L,rms}^2 + i_{R,rms}^2} = 321\,mA}$.

10. A He-Ne laser has a power output of 5.0mW in a beam of 1.00mm diameter. Find (a)the intensity of the beam, (b)the peak electric field in the beam and (c)the peak magnetic field in the beam.

(a)The intensity is the power per unit area, $S = \frac{P}{A} = \frac{5x10^{-3}}{\pi(0.0005)^2} = \underline{\underline{6370\,\frac{W}{m^2}}}$.

(b)The Poynting Vector is related to the peak field, $S_{av} = \frac{E_m^2}{2\mu_o c} \Rightarrow E_m = \sqrt{2\mu_o cS} = \underline{\underline{2190\,N/C}}$.

(c)The ratio of the peak fields is the speed of light, $B_m = \frac{E_m}{c} = \underline{\underline{7.30\,\mu T}}$.

Laws, Principles, Useful Relationships, and Other Information

Coulomb's Rule $\vec{F}_e = k\dfrac{q_1q_2}{r^2}\hat{r}$ Def'n of E-Field $\vec{E} \equiv \dfrac{\vec{F}}{q}$ E-Field for Point Charge $\vec{E} = k\dfrac{q}{r^2}\hat{r}$

Electric Potential $\Delta V = -\int \vec{E} \bullet d\vec{s}$ Def'n of Potential $\Delta U \equiv q\,\Delta V$ Potential of a Charge $V = k\dfrac{q}{r}$

Def'n of Capacitance $C \equiv \dfrac{Q}{V}$ Cap. of Parallel Plates $C = \varepsilon_o\dfrac{A}{d}$ Energy in C's $U = \dfrac{1}{2}\dfrac{Q^2}{C} = \dfrac{1}{2}CV^2$

Energy in E-Fields $u \equiv \dfrac{U}{vol} = \frac{1}{2}\varepsilon_o E^2$ B-Field $u_m = \dfrac{B^2}{2\mu_o}$ C's: Series $\dfrac{1}{C_s} = \sum\dfrac{1}{C_i}$ Parallel $C_p = \sum C_i$

Def'n of Current $I \equiv \dfrac{dQ}{dt}$ Dipoles: Torque $\vec{\tau} = \vec{p} \times \vec{E}$ or $\vec{\tau} = \vec{\mu} \times \vec{B}$ Energy $U = -\vec{p} \bullet \vec{E}$ or $U = -\vec{\mu} \bullet \vec{B}$

Def'n of Resistance $R = \rho\dfrac{\ell}{A}$ Ohm's Rule $V = IR$ Power $P = IV = \dfrac{V^2}{R} = I^2R$

R's: Series $R_s = \sum R_i$ Parallel $\dfrac{1}{R_p} = \sum\dfrac{1}{R_i}$ RC: Charge $q = CV\left(1 - e^{-t/RC}\right)$ discharge $q = CV_o e^{-t/RC}$

Force Between Wires $F_m = \dfrac{\mu_o}{2\pi}\dfrac{I_1I_2}{r}\ell$ Def'n of B-Field $\vec{F} \equiv I\vec{\ell} \times \vec{B}$ Force on a Charge $\vec{F} = q\vec{v} \times \vec{B}$

B-Field of Wire $B = \dfrac{\mu_o I}{2\pi r}$ Def'n of Magnetic Dipole $\vec{\mu} \equiv I\vec{A}$ Biot-Savart Rule $\vec{B} = \dfrac{\mu_o}{4\pi}\int\dfrac{Id\vec{s} \times \hat{r}}{r^2}$

Def'n of Flux $\Phi_B \equiv \int \vec{B} \bullet d\vec{A}$ or $\Phi_E \equiv \int \vec{E} \bullet d\vec{A}$ Def'n of Inductance $\mathcal{E} \equiv -L\dfrac{dI}{dt}$

Gauss's Law for E $\oint \vec{E} \bullet d\vec{A} = \dfrac{q}{\varepsilon_o}$ Faraday's Law $\oint \vec{E} \bullet d\vec{s} = -\dfrac{d\Phi_B}{dt}$ Gauss's Law for B $\oint \vec{B} \bullet d\vec{A} = 0$

Ampere's Law $\oint \vec{B} \bullet d\vec{s} = \mu_o\dfrac{dq}{dt} + \mu_o\varepsilon_o\dfrac{d\Phi_e}{dt}$ LC Circuit $Q = Q_m\cos(\omega t + \delta)$ where $\omega = \frac{1}{\sqrt{LC}}$

LR: "charging" $I = I_o\left(1 - e^{-\frac{R}{L}t}\right)$ "discharging" $I = I_o e^{-\frac{R}{L}t}$ Energy in Inductors $U_L = \frac{1}{2}LI^2$

LRC $Q = Q_m e^{-\frac{Rt}{2L}}\cos\omega_d t$ where $\omega_d = \sqrt{\dfrac{1}{LC} - \left(\dfrac{R}{2L}\right)^2}$ Transformers: $I_pV_p = I_sV_s$ and $\dfrac{V_p}{N_p} = \dfrac{V_s}{N_s}$

Def'n of Impedance $X \equiv \dfrac{\mathcal{E}_m}{I_m}$ Impedances: $X_R = R$ $X_C = \dfrac{1}{\omega C}$ $X_L = \omega L$

RLC Circuit: Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$ Phase Angle $\tan\phi = \dfrac{X_L - X_C}{R}$ Resonance Freq. $\omega_o = \dfrac{1}{\sqrt{LC}}$

EM Waves: $c = \dfrac{1}{\sqrt{\mu_o\varepsilon_o}}$ $E_m = cB_m$ $k \equiv \frac{2\pi}{\lambda}$ $\frac{\omega}{k} = \lambda f = c$

The Def'n of Poynting Vector $\vec{S} \equiv \frac{1}{\mu_o}\vec{E} \times \vec{B}$ Average $S_{av} = \dfrac{E_mB_m}{2\mu_o} = \dfrac{E_m^2}{2\mu_o c} = \dfrac{c}{2\mu_o}B_m^2$

Complete Absorption: Momentum Transfer $p = \dfrac{U}{c}$ Radiation Pressure $P = \dfrac{S}{c}$

Physical Constants: $e = 1.60\text{x}10^{-19}\,C$ $m_e = 9.11\text{x}10^{-31}\,kg$ $m_p = 1.67\text{x}10^{-27}\,kg$ $c = 3.00\text{x}10^{8}\,\frac{m}{s}$

$\mu_o = 4\pi\text{x}10^{-7}\,\frac{T\cdot m}{A}$ $k = 8.99\text{x}10^{9}\,\frac{N\cdot m^2}{C^2}$ $\varepsilon_o = 8.85\text{x}10^{-12}\,\frac{C^2}{N\cdot m^2}$