Name:____ Posting Code _____ _____ _____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded. *If you want your grade posted, put alphanumeric characters in the three spaces at the top right.*

1. Write down Maxwell's Equations. Name each one and explain what it means.

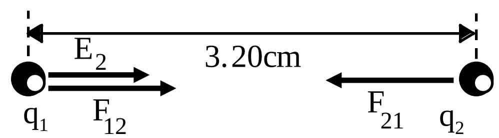
Gauss's Law for Electricity $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_o}$ Charges creates diverging electric fields.

Faraday's Law $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ Changing magnetic flux creates circulating electric fields.

Gauss's Law for Magnetism $\oint \vec{B} \cdot d\vec{A} = 0$ Their are no magnetic monopoles and magnetic fields never diverge.

Ampere's Law $\oint \vec{B} \cdot d\vec{s} = \mu_o \frac{dq}{dt} + \mu_o \varepsilon_o \frac{d\Phi_e}{dt}$ Current or changing electric fields create circulating magnetic fields.

2. The charge q_1=+12.0μC has a mass of 3.00g and the charge q_2=-8.0μC has a mass of 5.00g. At some instant in time they are a distance 3.20cm apart as shown at the right. Find (a)the electric field felt by the charge q_1, (b)the electric force on the charge q_1, (c)the electric force on the charge q_2 and (d)sketch the directions of these vectors in the drawing at the right.



(a)The field felt by q_1 is created by the point charge q_2,

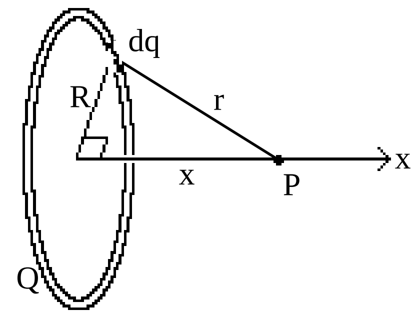
$E_2 = k\frac{q_2}{r^2} = (9.00\text{x}10^9)\frac{8.00\text{x}10^{-6}}{(0.0320)^2}$ $\boxed{E_2 = 7.03\text{x}10^7\text{N/C}}$

(b)The force on q_1 can be found from the definition of electric field,

$F_{12} = q_1E_2 = (12.0\text{x}10^{-6})(7.03\text{x}10^7)$ $\boxed{F_{12} = 844\text{N}}$

(c) The force on q_2 is equal and opposite to the force on q_1 according to Newton's Third Law, $\boxed{F_{21} = 844\text{N}}$

3. Find the potential due to a ring of total charge Q and radius R at the point P a distance x from the center of the ring along the axis as shown at the right.



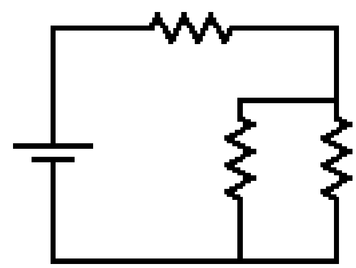
The potential due to the point charge dq is $dV = k\frac{dq}{r}$ $V = k\int\frac{dq}{r}$.

For all the dq's the r is constant so $V = \frac{k}{r}\int dq$.

The sum of all the dq's is just Q. Therefor $V = k\frac{Q}{r}$.

Writing r in terms of R and x gives the answer $\boxed{V = k\frac{Q}{\sqrt{R^2 + x^2}}}$

4. For the circuit shown the current through R_3 is measured to be 2.00mA. Find (a)the potential difference across each resistor, (b)the current through each resistor, (c)the current supplied by the battery and (d)the battery voltage. R_1=3.00kΩ, R_2=9.00kΩ and R_3=1.80kΩ.



V(V)	I(mA)	R(kΩ)
18.0	6.00	3.00
36.0	4.00	9.00
36.0	2.00	18.0
54.0	6.00	battery

(a)&(b)The voltage across R_3 can be found from Ohm's Rule,

$$V_3 = i_3R_3 = (2.00)(18.0) = 36.0V.$$

Since R_2 is in parallel with R_3 it must have the same voltage. The current through R_2 can now be found from Ohm's Rule,

$$i_2 = \frac{V_2}{R_2} = \frac{36.0}{9.00} = 4.00mA.$$

Using the junction theorem, the current through R_1 must equal the sum of the currents through R_2 and R_3,

$$i_1 = i_2 + i_3 = 4.00 + 2.00 = 6.00mA.$$

The voltage can be found from Ohm's Rule,

$$V_1 = i_1R_1 = (6.00)(3.00) = 18.0V.$$

(c)Since R_1 is in series with the battery, the current through both must be the same.
(d)Using the loop theorem, the battery voltage must equal the sum of the voltages on R_1 and R_2,

$$V = V_1 + V_2 = 18.0 + 36.0 = 54.0V.$$

5. In lab you measured the charge to mass ratio of an electron. Suppose your data indicated that the magnetic field was 700±20μT, the velocity of the electron was $3.3\pm0.3\text{x}10^7$m/s perpendicular to the field and the radius of curvature was 26±1cm. (a)Find the experimental value of e/m and (b)the uncertainty in this value.

(a)Using Newton's Second Law and the force on a moving charge in a magnetic field, $\Sigma F = ma \qquad evB = ma$.
Since the force is always perpendicular to the velocity, the motion is circular.

Using the centripetal acceleration, $evB = m\frac{v^2}{r} \qquad \frac{e}{m} = \frac{v}{rB} = \frac{3.3\text{x}10^7}{(0.26)(700\text{x}10^{-6})}$ $\boxed{\frac{e}{m} = 1.81\text{x}10^{11}\ \frac{C}{kg}}$.

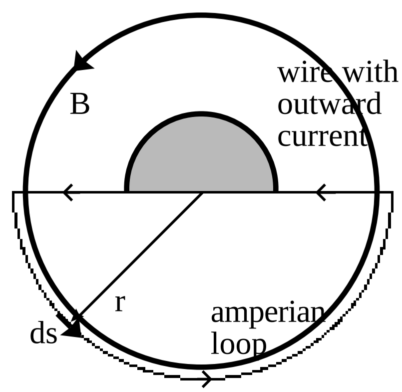
(b)Using the multiplication rule,

$$\frac{\Delta\frac{e}{m}}{\frac{e}{m}} = \sqrt{\left(\frac{\Delta v}{v}\right)^2 + \left(\frac{\Delta r}{r}\right)^2 + \left(\frac{\Delta B}{B}\right)^2} = \sqrt{\left(\frac{0.3}{3.3}\right)^2 + \left(\frac{1}{26}\right)^2 + \left(\frac{20}{700}\right)^2} = 0.10$$

$$\Delta\tfrac{e}{m} = 0.10\,\tfrac{e}{m} = (0.10)(1.81\text{x}10^{11}) = 0.2\text{x}10^{11}\ \tfrac{C}{kg}.$$

In summary, $\boxed{\frac{e}{m} = (1.8 \pm 0.2)\text{x}10^{11}\ \frac{C}{kg}}$.

6. A long straight wire of circular cross-section carries a uniformly distributed current of 3.00A and has a radius of 2.00cm. The amperian loop shown in the sketch at the right is a semi-circle of radius 5.00cm that passes through the center of the wire. For this amperian loop without using Ampere's Law, find (a)the magnetic field along the circular arc, (b)the line integral of B along this arc and (c)the line integral of B along the straight section. (d)Find the line integral around the entire amperian loop and explain how your result agrees with Ampere's Law.



(a)The magnetic field due to a long straight wire is, $B = \frac{\mu_o I}{2\pi r}$.

Along the arc, r is constant and so is the field,

$B = (2.00\text{x}10^{-7})\frac{3.00}{0.0500} \qquad \boxed{B = 12.0\mu T}$.

(b)The line integral of B can be found using the facts that B is along the arc and constant.

$$\int_{arc}\vec{B}\cdot d\vec{s} = \int_{arc}B\,ds = B\int_{arc}ds = B\pi r = \frac{\mu_o I}{2\pi r}\pi r = \tfrac{1}{2}\mu_o I = \tfrac{1}{2}(4\pi\text{x}10^{-7})(3.00)$$

$$\boxed{\int_{arc}\vec{B}\cdot d\vec{s} = 6\pi\text{x}10^{-7}\ T\ m = 1.88\text{x}10^{-6}\ T\ m}.$$

(c)Along the straight section the B field is perpendicular to the path so the line integral must be zero,

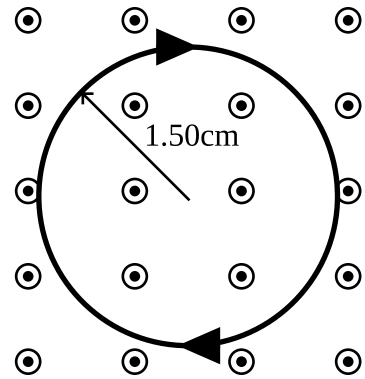
$$\boxed{\int_{line}\vec{B}\cdot d\vec{s} = 0}.$$

(d)The line integral around the entire amperian loop is,

$$\oint\vec{B}\cdot d\vec{s} = \int_{line}\vec{B}\cdot d\vec{s} + \int_{larc}\vec{B}\cdot d\vec{s} = 0 + \tfrac{1}{2}\mu_o I \qquad \boxed{\oint\vec{B}\cdot d\vec{s} = \tfrac{1}{2}\mu_o I}.$$

This agrees with the Ampere's Law which requires the line integral around the entire path to be proportional to the enclosed current which is one-half the total current.

7. The magnetic field through a 1000 turn circular loop of radius 1.50cm grows with time according to the equation $B = B_o(1+\frac{t}{T})$ where B_o=100μT and T=300s. Find the induced voltage in the loop at t=50.0s and show the direction of the induced current.



Starting with Faraday's Law, $\oint \vec{E}\cdot d\vec{s} = -N\frac{d\Phi_B}{dt}$ $\quad V = -N\frac{d\Phi_B}{dt}$

then using the definition of flux, $V = -N\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$.

Over the plane of the loop the field is constant and points along the area vector so,

$$V = -N\frac{d}{dt}BA = -N\frac{d}{dt}B_oA\left(1+\frac{t}{T}\right)$$

Doing the derivative and plugging in the values,

$$V = \frac{NB_oA}{T} = \frac{NB_o\pi r^2}{T} = \frac{(1000)(100\mu)\pi(0.0150)^2}{300} \quad \boxed{V = 236\text{x}10^{-9}\text{V}}.$$

The minus sign has been ignored because it refers to the direction of the current. The direction of the current flow can be found from Lenz's Rule. Since the field out of the page is increasing the current will be induced to create field into the page as shown.

8. The inductor you use in lab has an inductance of 680mH and a resistance of 135Ω. Find (a)the current through the inductor a long time after it is connected to a 1.50V battery and (b)the time it takes the current to reach half this value.

(a)After a long time the current has stopped changing and reached equilibrium. At this point the voltage is dropped across the resistance. Using Ohm's Rule,

$$V = IR \quad I_o = \frac{V}{R} = \frac{1.50}{135} \quad \boxed{I_o = 11.1\text{mA}}.$$

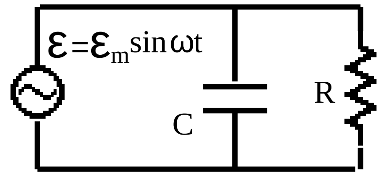
(b)The current builds up in an inductor according to, $I = I_o\left(1-e^{-\frac{R}{L}t}\right)$.

The time when the current reaches half the equilibrium value can be found from,

$$\tfrac{1}{2}I_o = I_o\left(1-e^{-\frac{R}{L}t}\right) \quad \tfrac{1}{2} = 1-e^{-\frac{R}{L}t} \quad e^{-\frac{R}{L}t} = \tfrac{1}{2} \quad -\frac{R}{L}t = \ln\tfrac{1}{2} = -\ln 2 \quad t = \frac{L}{R}\ln 2$$

$$t = \frac{0.680}{135}\ln 2 \quad \boxed{t = 3.49\text{ms}}.$$

9. A 15.0μF capacitor is connected in parallel with a 50.0Ω resistor and sine wave generator. (a)Find the ratio of the peak current through the resistor to the peak current through the capacitor at 60.0Hz and (b)60.0kHz.



Since the capacitor and the resistor are in parallel, they must have the same peak voltage. Using the definition of inductance,

$$V_{C,max} = V_{R,max} \qquad I_{C,max}X_C = I_{R,max}X_R \qquad \frac{I_{R,max}}{I_{C,max}} = \frac{X_C}{X_R}.$$

Using the impedances of a resistor and the capacitor,

$$\frac{I_{R,max}}{I_{C,max}} = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega RC} = \frac{1}{2\pi fRC}.$$

(a)Plugging in the numbers,

$$\frac{I_{R,max}}{I_{C,max}} = \frac{1}{2\pi(60.0)(50.0)(15.0\mu)} \qquad \boxed{\frac{I_{R,max}}{I_{C,max}} = 3.54}$$

(b)Plugging in the numbers for the higher frequency,

$$\frac{I_{R,max}}{I_{C,max}} = \frac{1}{2\pi(60.0k)(50.0)(15.0\mu)} \qquad \boxed{\frac{I_{R,max}}{I_{C,max}} = 0.00354}$$

As expected, there is more current through the resistor at low frequencies and less at high frequencies.

10. A laser emits pulses 2.00ns long with a power of $150x10^{12}$W at a wavelength of 1.25μm. Find (a)the energy in each pulse and (b)the speed of each pulse.

(a)Power is defined as the rate of flow of energy so, $P = \frac{dE}{dt} \qquad dE = Pdt \qquad E = Pt$ assuming the power is constant during the pulse. Putting in the values, $E = (150x10^{12})(2.00x10^{-9})$ $\boxed{E = 300kJ}$.

(b)The speed of all EM radiation is the speed of light, $\boxed{v = 3.00x10^8 \text{ m/s}}$.