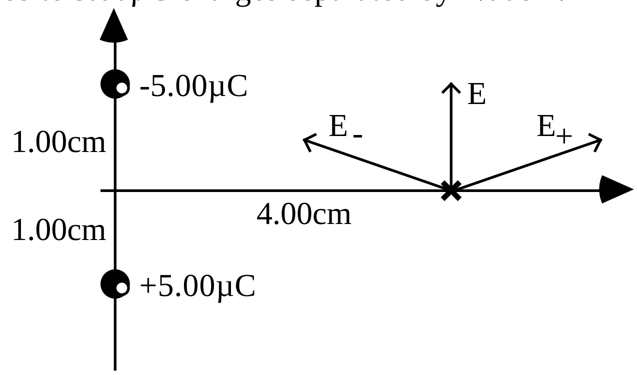
Physics 4B FINAL EXAM Chapters 23 - 34 Spring 1995

Name:____ Posting Code _____ _____ _____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded.

1. The dipole shown at the right consists of two equal but opposite 5.00μC charges separated by 2.00cm. Find the electric field 4.00cm from the center of the dipole perpendicular to the dipole axis.



The field due to a point charge the point charges are,

$\vec{E} = k\frac{q}{r^2}\hat{r} \qquad E_+ = E_- = k\frac{q}{r^2}$

The horizontal components will cancel leaving only the vertical components,

$E = E_+\sin\theta + E_-\sin\theta = 2k\frac{q}{r^2}\sin\theta$

where $r = \sqrt{4^2+1^2} = 4.12\text{cm}$ and $\sin\theta = \frac{1}{4.12} = 0.243$.

Finally,

$$\boxed{E = 2(9\text{x}10^9)\frac{5\text{x}10^{-6}}{0.0412^2}(0.243) = 1.29\text{x}10^7\ \text{N/C}}$$

2. A solid metal sphere of radius 5.00cm creates an electric field of $2.25\text{x}10^6$N/C at a distance of 20.0cm from its center. Describe the charge distribution on the sphere.

Applying Gauss's Law to an imaginary sphere 20.0cm in radius, $\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_o}$.

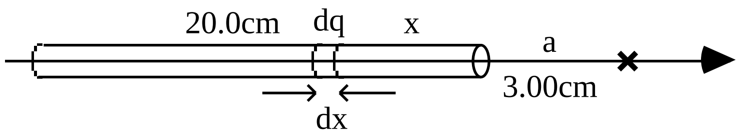
By the spherical symmetry of the problem, the field must be radial and constant on this imaginary sphere so,

$EA = \frac{q}{\varepsilon_o} \qquad q = \varepsilon_o EA = \varepsilon_o E4\pi r^2 = (8.85\text{x}10^{-12})(2.25\text{x}10^6)4\pi(0.200)^2$.

The charge on the sphere is, $\boxed{q = 10.0\mu\text{C}}$.

Using Gauss's Law and the same symmetry arguments for an imaginary sphere inside the metal sphere, $EA = \frac{q}{\varepsilon_o} \qquad q = \varepsilon_o EA$. However, the field inside a conductor is always zero, so the charge inside is also zero. Therefor, all the charge on the sphere is on the surface.

3. Find the potential due to a 20.0cm long rod that has a linear charge density of 2.00μC/m at a point 3.00cm from the end of the rod along the same axis as the rod.

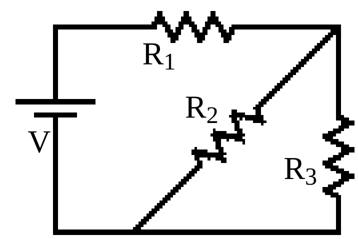


The potential due to the point charge dq is, $dV = k\frac{dq}{r}$.
The charge dq can be written in terms of the charge density, $dq = \lambda dx$.
The distance between dq and the point where we need the field is, $r = x + a$.

The potential is, $dV = k\frac{\lambda dx}{x+a}$ $\quad V = k\lambda\int_0^L \frac{dx}{x+a} = k\lambda \ln\left(\frac{L+a}{a}\right)$.

Plugging in the numbers, $\boxed{V = (9x10^{9})(2x10^{-6})\ln\left(\frac{23}{3}\right) = 3.67x10^{4}\,V}$.

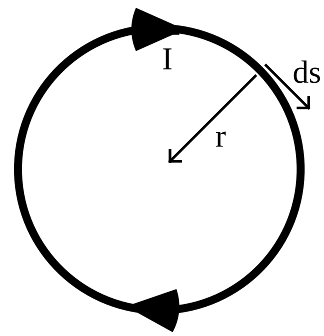
4. For the circuit shown find (a)the equivalent resistance, (b)the current supplied by the battery, (c)the power supplied by the battery, (d)the potential difference across each resistor, (e)the current through each resistor, and (f)the power consumed by each resistor. V=12.0V, R_1=1.00kΩ, R_2=3.00kΩ and R_3=6.00kΩ.



V(V)	I(mA)	R(kΩ)	P(mW)
4.00	4.00	1.00	16.0
8.00	2.67	3.00	21.3
8.00	1.33	6.00	10.7
battery 12.0	battery 4.00	equivalent 3.00	battery 48.0

(a)R_2 and R_3 are in parallel so $\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3}$ $\quad \frac{1}{R_p} = \frac{1}{3} + \frac{1}{6}$ $\quad R_p = 2.00k\Omega$.
R_1 is in series with R_p so $R_{eq} = R_1 + R_p = 3.00k\Omega$.
(b)The current from the battery can be found using Ohm's Rule, $V = iR$ $\quad i = \frac{V}{R_{eq}} = \frac{12}{3} = 4.00mA$.
(c)The power supplied by the battery is, $P = IV = (4)(12) = 48.0mW$.
(d)Since R_1 is in series with the battery, the current through it is the same as the battery. The voltage on R_1 can be found from Ohm's Rule. Using the loop theorem, the voltage on R_2 and R_3 must be equal and it must be the battery voltage less the voltage drop on R_1. From Ohm's Rule the current on R_2 and R_3can be found.
(e)The power used by a resistor is $P = IV = i^2R$.

5. Find the magnitude and direction of the magnetic field at the center of the coil shown at the right. The coil has 1000 turns, a radius of 3.00cm and carries 50.0mA of current.



Starting with the Biot-Savart Rule, $\vec{B} = \frac{\mu_o}{4\pi}\int \frac{I d\vec{s} \times \hat{r}}{r^2}$.

In this case, $d\vec{s} \times \hat{r}$ point <u>into the paper</u> and has a magnitude of ds.

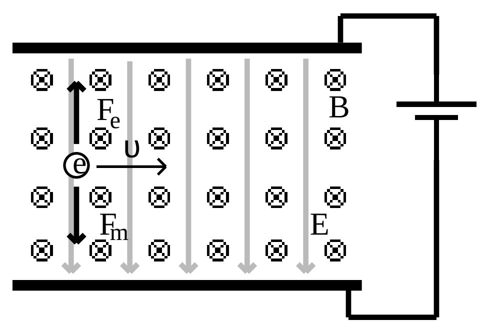
Since the radius and current are constant, $B = \frac{\mu_o I}{4\pi r^2}\int ds$.

The integral gives the circumference times the number of turns,

$B = \frac{\mu_o I}{4\pi r^2} 2\pi r N = \frac{\mu_o N I}{2r}$.

Putting in the numbers, $\boxed{B = \frac{(4\pi x 10^{-7})(1000)(0.05)}{2(0.03)} = 1.05 mT}$

6. A electron traveling at $3.00x10^5$m/s enters the crossed electric and magnetic fields shown at the right (the magnetic field in into the paper and the electric field is down the page). The magnetic field in this region is 3.00T. (a)Indicate the directions of the electric and magnetic forces on the electron. (b)Find the strength of the electric field required to keep the electron from being deflected.



(a)The electric force on an electron is opposite the E-field.
The magnetic force on an electron is given by the right hand rule, but remember that the charge is negative.

(b)Apply the Second Law, $\Sigma F = ma$ $\quad F_e - F_m = 0 \quad F_e = F_m$.

Using the definition of electric field and the force on a moving charge, $eE = evB \quad E = vB$.

Putting in the numbers, $\boxed{E = (3x10^5)(3) = 9.00x10^5 \frac{V}{m}}$.

7. An oscilloscope is used to measure the time it takes an inductor to lose half it's initial voltage when it is disconnected from a DC supply. This half-time is 3.50±.05ms and the resistance of the inductor is 135±1Ω. Find the inductance and its uncertainty.

The equation for the "discharge" of an inductor is, $I = I_o e^{-\frac{R}{L}t}$.
At this time, the current is half the initial current,

$$\frac{I}{I_o} = e^{-\frac{R}{L}t} \qquad \frac{1}{2} = e^{-\frac{R}{L}t_{1/2}} \qquad \ln\left(\frac{1}{2}\right) = -\frac{R}{L}t_{1/2} \qquad t_{1/2} = \frac{L}{R}\ln(2).$$

Solving for L and putting in the numbers, $L = \frac{Rt_{1/2}}{\ln(2)} = \frac{(135)(0.0035)}{\ln(2)} = 682\text{mH}$.

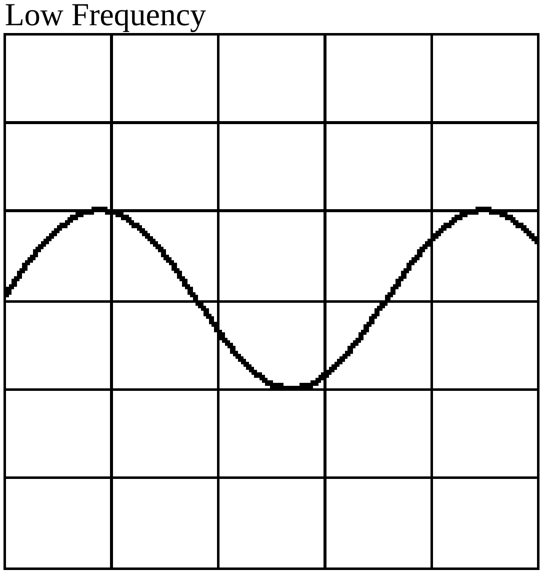
The uncertainty equation is from the multiplication rule,

$$\Delta L = L\sqrt{\left(\frac{\Delta R}{R}\right)^2 + \left(\frac{\Delta t_{1/2}}{t_{1/2}}\right)^2} \qquad \Delta L = (682)\sqrt{\left(\frac{1}{135}\right)^2 + \left(\frac{5}{350}\right)^2} = 11\text{mH}.$$

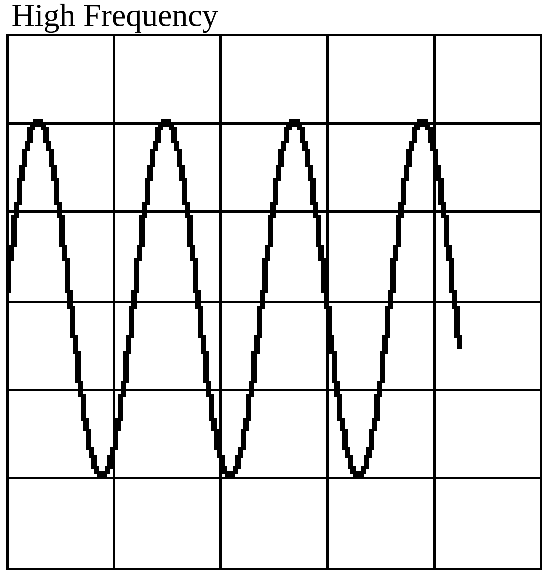
Using proper significant figures, $\boxed{L = 680 \pm 10\text{mH}}$.

8. A coiling is spinning in a constant magnetic field. The output of the coil is connected to an oscilloscope. Sketch the scope output for a low frequency and a high frequency. Explain your reasoning using the appropriate physical concepts.

Low Frequency

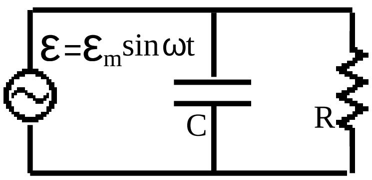


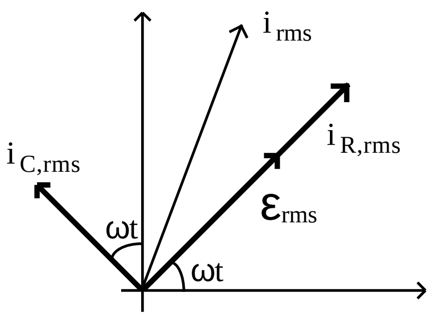
High Frequency



Faraday's Law states that the induced voltage in a loop is proportional to the rate of change of the magnetic flux. The flux in the loop changes more rapidly when the coil spins faster.

9. A 20.0μF capacitor is connected in parallel with a 50.0Ω resistor and a 60.0Hz - 15.0V rms power supply. (a)Draw the phasor diagram for this circuit. Find the rms voltage across (b)the power supply, (c)the capacitor and (d)the resistor. Find the rms current (e)through the capacitor, (f)through the resistor and (g)provided by power supply.





(a)
Note that since the circuit elements are in parallel, they all have the same voltage phasor.

(b)This is given to be $\boxed{\varepsilon = 15.0\text{V}}$.

(c)Since the capacitor is in parallel with the power supply the loop theorem requires the voltages to be equal, $\boxed{V_{C,rms} = 15.0\text{V}}$.

(d)Since the resistor is also in parallel with the power supply the loop theorem requires, $\boxed{V_{R,rms} = 15.0\text{V}}$.

(e)Using the definition of impedance and the impedance of a capacitor,

$$V_{C,rms} = i_{C,rms}\chi_C \qquad i_{C,rms} = \frac{V_{C,rms}}{\chi_C} = 2\pi f C V_{C,rms}$$

Putting in the numbers, $\boxed{i_{C,rms} = 2\pi(60)(20\text{x}10^{-6})(15) = 113\text{mA}}$.

(f)Using the definition of impedance and the impedance of a resistor,

$$V_{R,rms} = i_{R,rms}\chi_R \qquad i_{R,rms} = \frac{V_{R,rms}}{\chi_R} = \frac{V_{R,rms}}{R}$$

Putting in the numbers, $\boxed{i_{R,rms} = \frac{15}{50} = 300\text{mA}}$.

(g)From the phasor diagram, $\boxed{i_{rms} = \sqrt{i_{C,rms}^2 + i_{R,rms}^2} = 321\text{mA}}$.

10. Find the peak electric and magnetic fields in a light waves that reach us from the sun. For sunlight, the average wavelength is 550nm and the average intensity is 1000W/m^2.

The average intensity is the Poynting Vector, $S_{av} = \frac{E_m^2}{2\mu_o c} \qquad E_m = \sqrt{2\mu_o c S_{av}}$.

Putting in the numbers, $\boxed{E_m = \sqrt{2(4\pi\text{x}10^{-7})(3\text{x}10^8)(1000)} = 868\frac{\text{V}}{\text{m}}}$.

The ratio of the peak fields is, $\boxed{B_m = \frac{E_m}{c} = \frac{868}{3\text{x}10^8} = 2.89\mu\text{T}}$.