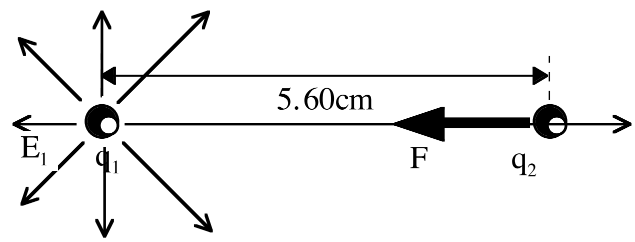
Name:________________________________________ Posting Code _____ _____ _____

Solve the following problems in the space provided. Use the back of the page if needed. Each problem is worth 10 points. You must show your work in a logical fashion starting with the correctly applied physical principles which are on the last page. Your score will be maximized if your work is easy to follow because partial credit will be awarded. *If you want your grade posted, put alphanumeric characters in the three spaces at the top right.*

1. The charge $q_1$=+10.0$\mu$C and the charge $q_2$=-12.0$\mu$C. At some instant in time they are a distance 5.60cm apart as shown at the right. (a)Sketch the electric field of the charge $q_1$ in the drawing at the right. (b)Find the magnitude of electric field felt by the charge $q_2$. (c)Find the electric force on the charge $q_2$ and (d)sketch the direction of the force vector in the drawing at the right.



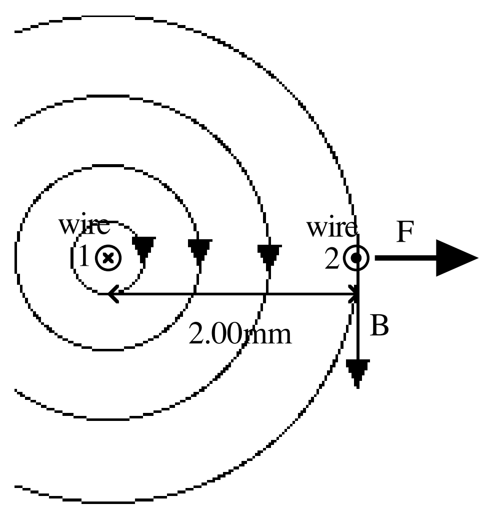

(b)The electric field felt by $q_2$ is the field created by the point charge $q_1$,

$$E_1 = k\frac{q_1}{r^2} = (9.00\text{x}10^9)\frac{10.0\text{x}10^{-6}}{(0.0560)^2} \Rightarrow \boxed{E_1 = 2.87\text{x}10^7 \tfrac{N}{C}}$$

(c)Using the definition of electric field,

$$E_1 = \frac{F}{q_2} \Rightarrow F = q_2E_1 = (12.0\text{x}10^{-6})(2.87\text{x}10^7) \Rightarrow \boxed{F = 344\text{N}}$$

2. The sketch at the right shows two wires that are perpendicular to the page. Wire 1 carries a current of 160mA into the page and wire 2 carries a current of 120mA out of the page. The wires are 2.00mm apart, 10.0$\mu$m in diameter and 3.20m long. (a)Sketch the magnetic field of wire 1 in the drawing at the right. (b)Find the magnitude of magnetic field felt by wire 2. (c)Find the magnetic force on wire 2 and (d)sketch the direction of the force vector in the drawing at the right.




(b)The magnetic field felt by wire 2 is the field created by the long straight wire 1,

$$B_1 = \frac{\mu_o I_1}{2\pi r} = \frac{(2.00\text{x}10^{-7})(0.160)}{0.00200} \Rightarrow \boxed{B_1 = 16.0\mu\text{T}}$$

(c)Using the definition of magnetic field,

$\vec{F} = I_2\vec{\ell} \times \vec{B}_1 \Rightarrow F = I_2\ell B_1 = (0.120)(3.20)(16.0\mu) \Rightarrow \boxed{F = 6.14\mu N}$

3. For each situation described below write the Maxwell Equation that describes or explains the field. Name the equation and indicate which terms on the right hand side are relevant and which can be ignored.

(a)The electric field due to a stationary point charge:

Gauss's Law for Electricity $\oint \vec{E} \bullet d\vec{A} = \frac{q}{\varepsilon_o}$. Charges create diverging electric fields.

(b)The magnetic field due to a current carrying wire:

Ampere's Law $\oint \vec{B} \bullet d\vec{s} = \mu_o \frac{dq}{dt} + \mu_o \varepsilon_o \frac{d\Phi_e}{dt}$ . The first term is the current and the second is zero in this case.

(c)The loop theorem:

Faraday's Law $\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$. Assuming there is no changing magnetic flux the right side is zero.

(d)The voltage induced in a coil when it is removed from a magnetic field:

Faraday's Law $\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$. The changing magnetic flux creates a voltage.

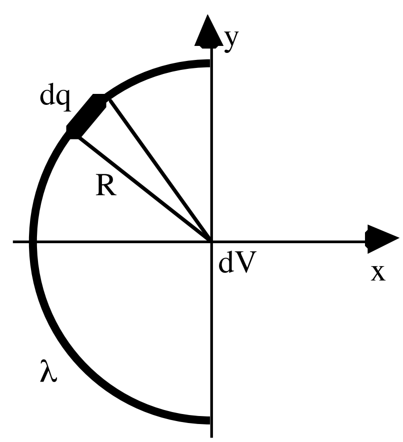
(e)The electric field between the plates of a charging capacitor:

Gauss's Law for Electricity $\oint \vec{E} \bullet d\vec{A} = \frac{q}{\varepsilon_o}$. The charges on the plate create the electric field.

(f)The magnetic field between the plates of a charging capacitor:

Ampere's Law $\oint \vec{B} \bullet d\vec{s} = \mu_o \frac{dq}{dt} + \mu_o \varepsilon_o \frac{d\Phi_e}{dt}$ . There is no current between the plates, so the first term is zero and the changing electric flux creates the magnetic field.

4. Find the potential at the origin due to a semicircular arc of radius R with a linear charge density λ(C/m).




The potential due to the point charge dq is:

$$dV = k\frac{dq}{R}$$

There is no need to worry about components for potential because it isn't a vector. The total potential is,

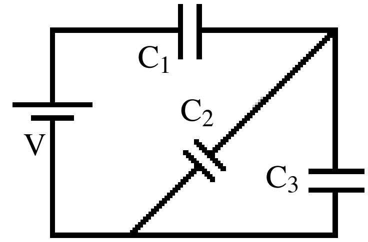
$$V = \int k\frac{dq}{R} = k\frac{1}{R}\int dq$$

Summing the total charge, $V = k\frac{Q}{R}$.

The total charge will be the linear charge density times the length of the arc,

$$V = k\frac{\lambda\pi R}{R} \Rightarrow \boxed{V = k\pi\lambda = \frac{\lambda}{4\varepsilon_o}}$$

5. The charge on $C_3$ shown in the circuit at the right is 8.00$\mu$C. Find (a)the potential difference across each capacitor, (b)the charge on each capacitor and (c)the voltage of the battery ($C_1$=4.00$\mu$F, $C_2$=6.00$\mu$F and $C_3$=12.0$\mu$F).




| Q($\mu$C) | C($\mu$F) | V(V) |
|---|---|---|
| 12.0 | 4.00 | 3.00 |
| 4.00 | 6.00 | 0.667 |
| 8.00 | 12.00 | 0.667 |

$V_{battery} = 3.67V$

The voltage across $C_3$ can be found using the definition of capacitance,

$$V_3 = \frac{Q_3}{C_3} = \frac{8}{12} = 0.667\text{V}.$$

Since $C_3$ is in parallel with $C_2$, they both have the same potential difference. Now, the charge on $C_2$ can be found,

$$Q_2 = C_2V_2 = (6)(0.667) = 4.00\mu\text{C}.$$

Since $C_1$ is in series with the combination of $C_2$ and $C_3$, the charge on $C_1$ must equal the sum of the charges on $C_2$ and $C_3$.

$$Q_1 = Q_2 + Q_3 = 4 + 8 = 12.0\mu\text{C}$$

The voltage across $C_1$ is,

$$V_1 = \frac{Q_1}{C_1} = \frac{12}{4} = 3.00\text{V}$$

By the loop theorem, the voltage on the battery should be the sum of the voltage on $C_1$ and either $C_2$ or $C_3$.

$$V = V_1 + V_3 = 3 + 0.667 = 3.67\text{V}$$

6. In the lab you conducted the e/m experiment by accelerating electrons with a potential difference of 2500V and sending them into a magnetic field which was adjusted so that the radius of curvature was 26.0cm. Find (a)the speed of the electrons and (b)the strength of the magnetic field. Some data you will need is on the last page.

(a)The Law of Conservation of Energy requires the electric potential energy lost by the electron equal the kinetic energy it gains,

$$\Delta U = -\Delta K \Rightarrow eV = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60\text{x}10^{-19})(2500)}{9.11\text{x}10^{-31}}} \Rightarrow \boxed{v = 2.96\text{x}10^7\,\text{m/s}}.$$

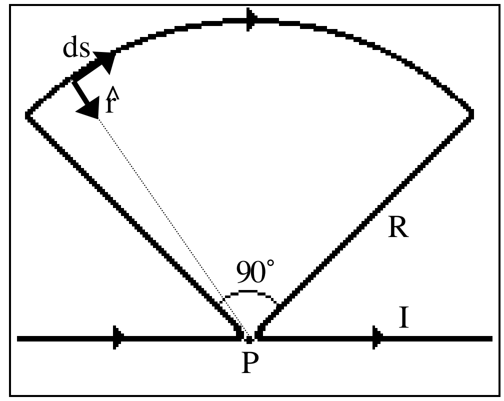
(b) When the electron enters the magnetic field, Newton's Second Law requires,

$$\Sigma F = ma \Rightarrow evB = ma.$$

Since the motion is circular the acceleration is centripetal,

$$evB = m\frac{v^2}{r} \Rightarrow B = \frac{mv}{er} = \frac{(9.11\text{x}10^{-31})(2.96\text{x}10^7)}{(1.60\text{x}10^{-19})(0.260)} \Rightarrow \boxed{B = 650\,\mu\text{T}}.$$

7. Find the magnetic field at the point P caused by the current I in the 90° circular arc of radius R.




Use the Biot Savart Rule $\vec{B} = \frac{\mu_o}{4\pi}\int \frac{I d\vec{s} \times \hat{r}}{r^2}$.

Since $d\vec{s}$ and $\hat{r}$ are parallel for all the straight sections of the wire, these sections create no magnetic field at the point P. The field at P is due only to the circular arc along which $d\vec{s}$ and $\hat{r}$ are perpendicular so $\vec{B}$ points into the page and

$$|d\vec{s} \times \hat{r}| = (ds)(1)\sin 90° = ds .$$

The magnitude of the field is,

$$B = \frac{\mu_o}{4\pi}\int \frac{I ds}{R^2} = \frac{\mu_o I}{4\pi R^2}\int ds = \frac{\mu_o I}{4\pi R^2} R\left(\frac{\pi}{2}\right) = \underline{\underline{\frac{\mu_o I}{8R}}} .$$

8. An RL circuit contains a 100±1Ω resistor. The time for the voltage on the resistor to drop from 1.50V down to 0.750V is 175±2$\mu$s. Find (a)the inductance of the circuit and (b)the uncertainty in the inductance. Assume the voltages are exact. The manufacturer claims the inductance is 30.0mH within 20%. (c)Which number should you use for the inductance, the experimental or the manufacturer's? Explain.

(a)For a "discharging inductor" $I = I_o e^{-\frac{R}{L}t}$.

Using Ohm's Rule, $V = IR = I_o R e^{-\frac{R}{L}t} = V_o e^{-\frac{R}{L}t}$.

Solving for the inductance, $L = \frac{Rt}{\ln\left(\frac{V_o}{V}\right)} = \frac{(100)(175\text{x}10^{-6})}{\ln(2)} \Rightarrow L = 25.2\text{mH}$

(b)The uncertainty equation is from the multiplication rule is,

$$\Delta L = L\sqrt{\left(\frac{\Delta R}{R}\right)^2 + \left(\frac{\Delta t}{t}\right)^2} \Rightarrow \Delta L = (25.2)\sqrt{\left(\frac{1}{100}\right)^2 + \left(\frac{2}{175}\right)^2} = 0.4\text{mH} .$$

Using proper significant figures, $\boxed{L = 25.2 \pm 0.4\text{mH}}$.

(c)While both numbers are in agreement, the experimental value is the one to use because it has a smaller uncertainty.

9. An ideal 280mH inductor is connected to a 60.0Hz - 15.0V rms power supply. Find the rms current through the inductor.

The impedance of an inductor is, $X_L = \omega L = 2\pi f L$.

Using the definition of impedance, $X \equiv \frac{V}{I} \Rightarrow I_{rms} = \frac{V_{rms}}{X_L} = \frac{V_{rms}}{2\pi f L} = \frac{15.0}{2\pi(60.0)(0.280)} \Rightarrow \boxed{I_{rms} = 142\,\text{mA}}$

10. Find the total power radiated by the sun assuming that $1400 W/m^2$ is the intensity of the sunlight reaching Earth. The sun is $1.50 x 10^{11}$m away and the radius of Earth is $6.37 x 10^6$m.

Since the sun radiates EM waves uniformly in all directions this intensity will be the same over the surface area of a sphere that is $1.50 x 10^{11}$m away.

Using the definition of power and the definition of intensity, $I \equiv \frac{dU}{Adt} = \frac{P}{A} = \frac{P}{4\pi r^2}$.

Solving for the power, $P = I 4\pi r^2 = (1400)(4\pi)(1.50 x 10^{11})^2 \Rightarrow \boxed{P = 3.96 x 10^{26} W}$.