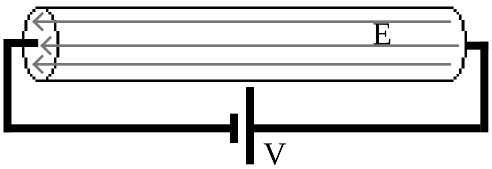
Chapter 27 - Current and Resistance

Problem Set #6 - due:
Ch 27 - 1, 8, 13, 19, 20, 35, 47, 54



When an potential difference is applied to a conductor an electric field is created inside. Immediately the free charges begin to flow to cancel the field. It is this flow of charge that we will study.

Lecture Outline
1. The Definitions of Current and Current Density
2. Resistivity and Ohm's Rule
3. Energy Transfer in Circuits

<u>1. The Definitions of Current and Current Density</u>

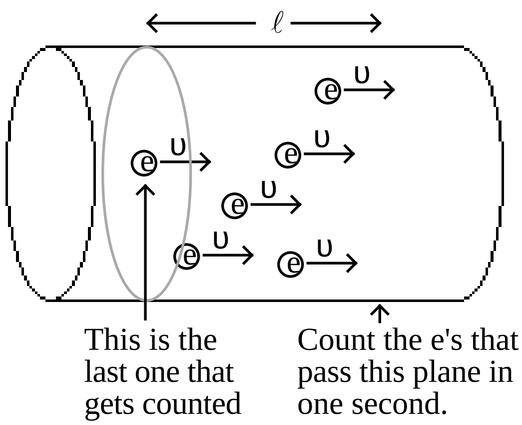
Current: The rate at which charge flows.

$$I \equiv \frac{dQ}{dt} \quad \text{The Definition of Current}$$

The units of current are: $[I] = \frac{[Q]}{[t]} = \frac{1 \text{ Coulomb}}{1 \text{ second}} \equiv 1 \text{ Ampere } = 1 \text{ Amp } = 1\text{A}$

> *<u>Example 1:</u> Find the number of electrons that pass a point in a wire carrying 1.00A during 1.00s.*
> Using the definition of current,
> $$I \equiv \frac{dQ}{dt} \qquad \int_0^{Ne} dQ = I\int_0^t dt \qquad Ne = It \qquad N = \frac{(1.00\text{A})\ (1.00\text{s})}{1.60\text{x}10^{-19}\text{C}} = 6.25\text{x}10^{18} \text{electrons}$$

Now let's ask the seemingly harmless question, how fast are the electrons going?



The speed of the electrons can be written as, $\upsilon = \frac{\ell}{t}$. The time can be found using the definition of current as in example 1,

$$I \equiv \frac{dQ}{dt} \qquad \int_0^{Ne} dQ = I\int_0^t dt \qquad Ne = It \qquad t = \frac{Ne}{I}.$$

Now the speed becomes, $\upsilon = \frac{\ell I}{Ne} = \frac{\ell A}{N} \frac{I}{A} \frac{1}{e}$

Define the free electron density and the current density,

$$n \equiv \frac{N}{vol} \quad \text{The Definition of Free Electron Density}$$

$$j \equiv \frac{I}{A} \quad \text{The Definition of Current Density}$$

In terms of these quantities, $\upsilon = \frac{j}{ne}$ $\quad j = ne\upsilon$. This speed is called "the drift velocity".

$$j = ne\upsilon \quad \text{Drift Velocity}$$

<u>*Example 2:*</u> *Find the average speed of the electrons in example 1. Assume the wire is made of copper and has a 1.00mm radius.*

First we need to estimate the free electron density for copper. Assuming that each copper atom contributes one free electron to the metal, we can find the free electron density from the mass density and the molecular weight:

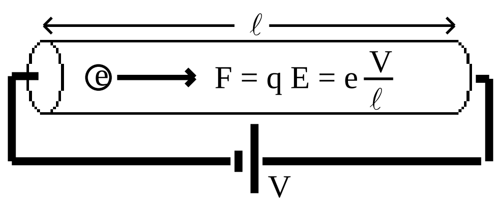
$$n = \left(9.00\text{x}10^3 \frac{\text{kg Cu}}{\text{m}^3}\right)\left(\frac{6.02\text{x}10^{23}\text{atoms}}{0.064\text{kg Cu}}\right)\left(\frac{1 \text{ free electron}}{\text{atom}}\right) = 8.47\text{x}10^{28}\,\frac{\text{electrons}}{\text{m}^3}.$$

Using the definition of current density, $j \equiv \frac{I}{A} = \frac{I}{\pi r^2} = \frac{1.00\text{A}}{\pi(0.001)^2} = 3.18\text{x}10^5\,\frac{\text{A}}{\text{m}^2}$. Applying the equation for drift velocity,

$$j = ne\upsilon \qquad \upsilon = \frac{j}{ne} = \frac{3.18\text{x}10^5}{\left(8.47\text{x}10^{28}\right)\left(1.60\text{x}10^{-19}\right)} = 2.35\text{x}10^{-5}\,\frac{\text{m}}{\text{s}} = \underline{\underline{0.0235\,\frac{\text{mm}}{\text{s}}}}.$$

This velocity is so small that it is hard to understand why a light bulb goes on almost instantly when the switch is flipped. There is something we haven't accounted for in our model of charge flow in conductors.

2. Resistivity and Ohm's Rule



According to this model of current flow the charge will have a constant acceleration. Using Newton's Second Law, $\sum F = ma \qquad a = \frac{F}{m} = \frac{qE}{m} = \frac{eV}{m\ell}$. If the acceleration is constant, the speed will increase indefinitely. This contradicts the concept of a drift velocity.

We must include collisions between the electrons and the Cu atoms. If we call τ the average time between collisions, the average velocity of the electrons will roughly be $\bar{\upsilon} \approx a\tau$. Using the acceleration above and the equation for drift velocity, $\frac{j}{ne} = \frac{eV}{m\ell}\tau \qquad \frac{I}{neA} = \frac{eV}{m\ell}\tau \qquad V = I\left(\frac{m}{\tau ne^2}\right)\left(\frac{\ell}{A}\right)$.

Define the resistivity as,

$\rho \equiv \frac{m}{\tau ne^2}$	The Definition of Resistivity

Notice that ρ is dependent on microscopic properties of the conducting material. The resistivity is difficult to calculate from these numbers, but it is not hard to measure. You will find tables of resistivity values. Sometimes the conductivity is tabulated. Conductivity, σ, is the reciprocal of the resistivity, $\sigma \equiv \frac{1}{\rho}$.

Now, $V = I\left(\rho\frac{\ell}{A}\right)$. Define resistance as,

$$R \equiv \rho\frac{\ell}{A} \quad \text{The Definition of Resistance}$$

Resistance takes into account the geometrical properties of the material. Finally we have Ohm's Rule:

$$V = IR \quad \text{Ohm's Rule}$$

The units of resistance are: $[R] = \frac{[V]}{[I]} = \frac{1\text{ Volt}}{1\text{ Amp}} \equiv 1\text{ Ohm} = 1\ \Omega$

Example 3: A 50.0m extension cord is made of copper wire that is 2.54mm in diameter. It carries a current of 25.0A. Find (a)the resistance of the extension cord and (b)the voltage drop in the cord.

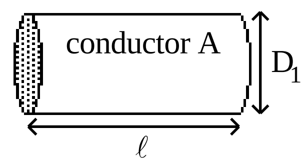
(a)Using the definition of resistance and the known resistivity of copper,

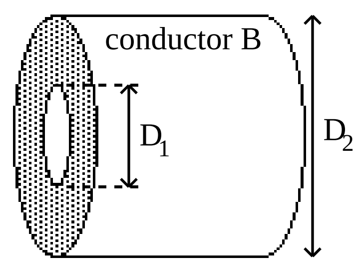
$$R \equiv \rho\frac{\ell}{A} = \rho\frac{\ell}{\frac{1}{4}\pi D^2} = \frac{4\rho\ell}{\pi D^2} = \frac{4(1.69\text{x}10^{-8})(100)}{\pi(0.00254)^2} = \underline{\underline{0.334\Omega}}.$$

Note that the length must be doubled because the cord has two wires. One carries current to the device at the end and one that returns the current to the source.

(b)Use Ohm's Rule to find the potential drop, $V = IR = (25.0)(0.334) = \underline{\underline{8.34V}}$.

Example 4: Two conductors of the same material and length have different resistances. Conductor A is a solid 1.00mm diameter wire. Conductor B is a tube of inner diameter 1.00mm and outer diameter 2.00mm. Find the ratio of the resistances of conductor A to conductor B.

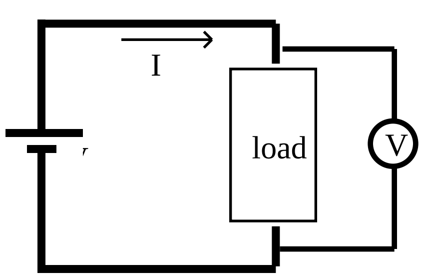




From the definition of resistance, $R_A \equiv \rho\frac{\ell}{A_A}$ and $R_B \equiv \rho\frac{\ell}{A_B}$.

The ratio is $\frac{R_A}{R_B} \equiv \frac{\rho\frac{\ell}{A_A}}{\rho\frac{\ell}{A_B}} = \frac{A_B}{A_A} = \frac{\frac{1}{4}\pi(D_2^2 - D_1^2)}{\frac{1}{4}\pi D_1^2} = \frac{2^2 - 1^2}{1^2} = \underline{\underline{3.00}}$

3. Energy Transfer in Circuits



By definition current is the number of charges per second and voltage is the amount of energy carried by each charge. The product of voltage and current must be the amount of energy per second. This is the power.

$$P = IV$$

Using Ohm's Rule, power can be written without V or without I:

$$P = IV = \frac{V^2}{R} = I^2R \quad \text{Electric Power}$$

<u>Example 5:</u> The extension cord of example 3 is connected to a 110V source. Find the (a)power supplied by the source, (b)power lost in the cord and (c)power supplied to the load.

(a)The electrical power supplied is $P = IV = (25.0A)(110V) = \underline{\underline{2750W}}$.

(b)The power lost in the cord can be found from the voltage drop,
$P = IV = (25.0A)(8.34V) = \underline{\underline{209W}}$.

The resistance of the cord could be used instead, $P = I^2R = (25.0A)^2(0.334\Omega) = \underline{\underline{209W}}$.

(c)By the Law of Conservation of Energy,
$P_{sup\ ply} = P_{cord} + P_{load} \qquad P_{load} = P_{supply} - P_{cord} = 2750 - 209 = \underline{\underline{2540W}}$.

Do you know why extension cords are rated for maximum length?

Explain why power lines use high voltage instead of high current.

Chapter 27 - Summary

The Definition of Current $I \equiv \frac{dQ}{dt}$

The Definition of Free Electron Density $n \equiv \frac{N}{vol}$

The Definition of Current Density $j \equiv \frac{I}{A}$

Drift Velocity $j = ne\upsilon$

The Definition of Resistivity $\rho \equiv \frac{m}{\tau ne^2}$

The Definition of Resistance $R \equiv \rho\frac{\ell}{A}$

Ohm's Rule $V = IR$

Electric Power $P = IV = \frac{V^2}{R} = I^2R$