Chapter 29 - Magnetic Fields

Problem Set #8 - due:
Ch 29 - 2, 5, 12, 14, 21, 30, 34, 37, 41, 45, 46, 49, 55, 60, 64, 69

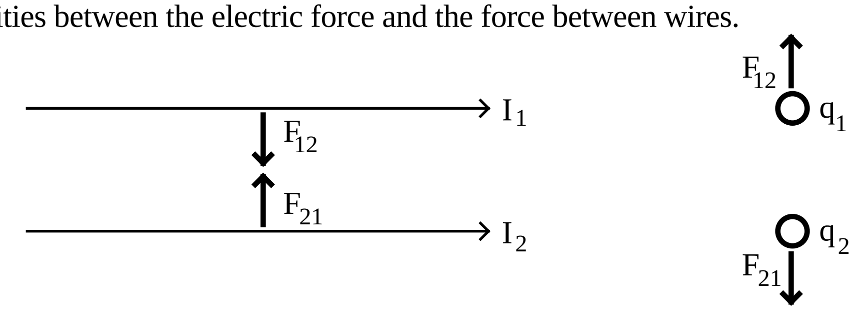
It turns out the hardest thing to understand about magnetism is a simple magnet. We will start by studying the force on a current caused by a magnet field. We'll wait until next chapter to figure out where the magnetic field comes from.

Lecture Outline
1. The Force Between Currents
2. The Definition of the Magnetic Field
3. The Magnetic Force on a Moving Charge
4. Current Loops in a Constant Field
5. Magnetic Devices

1. The Force Between Currents

Current Balance

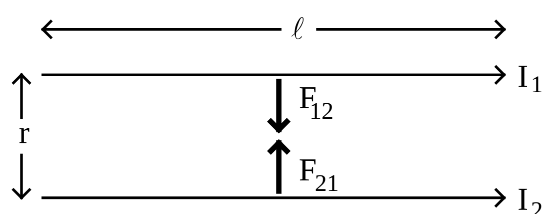
There are similarities between the electric force and the force between wires.



Could the force between wires just be the electric force? It would appear not because:
1)Like currents attract and opposite repel, exactly the reverse of the electric force.
2)The current carrying wires are electrically neutral. They exert no force on a single charge.

It would seem that we must treat the force between current carrying wires as a new force called the "magnetic force." It must be noted that, in fact, this force is electrical in nature and Einstein's Theory of Relativity explains the connection between electricity and magnetism.

We need to establish the force law (analogous to Coulomb's Rule or Newton's Law of Universal Gravitation) for this "new" force of magnetism.



Newton's Third Law requires $F_{12} = F_{21}$

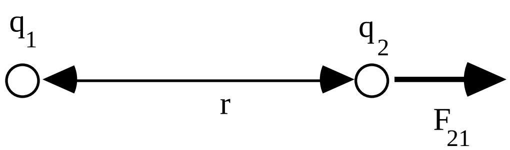
Guess: $F_{12} \quad I_1$, $F_{12} \quad I_2$, $F_{12} \quad \frac{1}{r}$ and $F_{12} \quad \ell$.

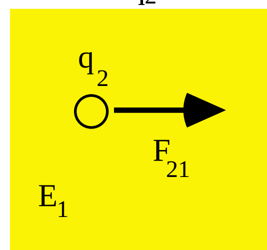
$$F_m = \frac{\mu_o}{2\pi}\frac{I_1 I_2}{r}\ell \quad \text{The Force Between Current Carrying Wires}$$

where the constant $\mu_o \equiv 4\pi x 10^{-7}\frac{N}{A^2}$.

2. The Definition of the Magnetic Field

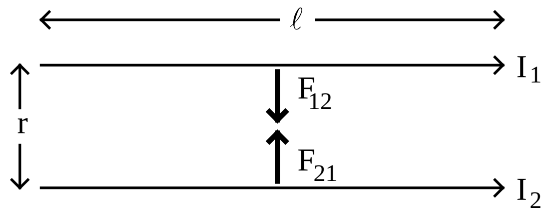
Recall the way we defined the electric field. Instead of thinking of q_1 exerting the force on q_2, we think of q_1 creating a field and the field exerting the force on q_2.

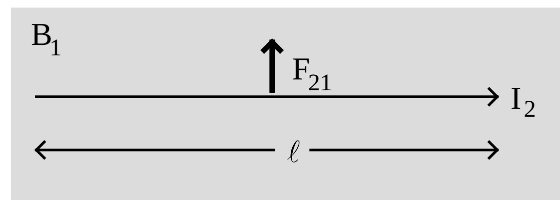




where $F_{21} = q_2 E_1$.

We can do the same thing with the magnetic field, B.





where $F_{21} = I_2 \ell B_1$.

To incorporate the vector nature of forces we need to pick a direction for the magnetic field. Since $\vec{F}$ is up and $\vec{\ell}$ is in the horizontal plane along the wire, it is most convenient to choose $\vec{B}$ into the paper. Now we can define the magnetic field vector,

$$\vec{F} \equiv I\vec{\ell} \times \vec{B} \quad \text{The Definition of the Magnetic Field}$$

Note the units: $[F] = [I][\ell][B] \qquad [B] = \dfrac{[F]}{[I][\ell]} = \dfrac{N}{A\ m}$

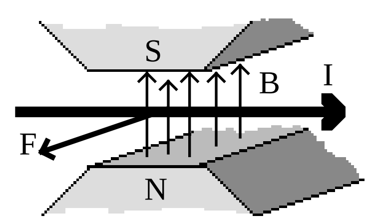
It is convenient to define a new unit $\dfrac{1\ N}{A\ m} \equiv 1\ \text{Tesla} = 1T$.

A second common unit is $1\ \text{Gauss} = 1G = 10^{-4}T$.

Currents are the source of the magnetic field. We will discuss this in detail in the next chapter. This chapter will focus on the effect of an applied magnetic field.

Wire & Magnet Demonstration - go over definition of magnetic field and r.h.r.

Example 1: An 0.100T magnet has a field that points upward. The pole faces have a 2.00cm diameter. Find the force on a 5.00A current flowing eastward.



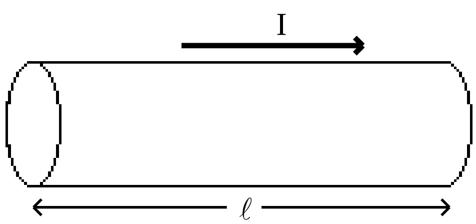
Use the definition of the magnetic field $\vec{F} \equiv I\vec{\ell} \times \vec{B}$.

Since the wire is perpendicular to the magnetic field $F = I\ell B$.

Since the pole faces are 2.00cm in diameter this is the length of the wire that feels the field. $F = (5.00)(0.0200)(0.100) = \underline{\underline{0.0100N}}$.

The force is southward by the right hand rule.

3. The Magnetic Force on a Moving Charge



The force on a current carrying wire is $\vec{F} \equiv I\vec{\ell} \times \vec{B}$. The current is composed of individual charges. We want to know the magnetic force on a single charge. Recall the definition of current density,

$$j \equiv \frac{I}{A} \qquad I = jA \qquad \vec{F} = jA\vec{\ell} \times \vec{B}.$$

Using the expression for the drift velocity,

$$j = nq\upsilon \qquad \vec{F} = nq\upsilon A\vec{\ell} \times \vec{B}.$$

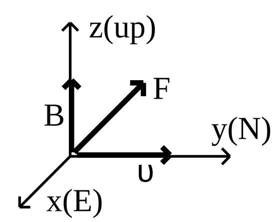
Inserting the free electron density $n \equiv \frac{N}{vol} = \frac{N}{A\ell}$ $\quad \vec{F} = \frac{N}{A\ell} q\upsilon A\vec{\ell} \times \vec{B} = Nq\upsilon\hat{\ell} \times \vec{B} = Nq\vec{\upsilon} \times \vec{B}$.

The force on a single charge is then,

$$\boxed{\vec{F} = q\vec{\upsilon} \times \vec{B} \quad \text{The Magnetic Force on a Charged Particle}}$$

Example 2: *A 2.00keV electron is fired northward into a uniform upward 0.100T magnetic field. (a)Find the force on the electron and (b)Describe its motion.*

(a)The definition of kinetic energy $K = \frac{1}{2}m\upsilon^2 \qquad \upsilon = \sqrt{\frac{2K}{m}}$



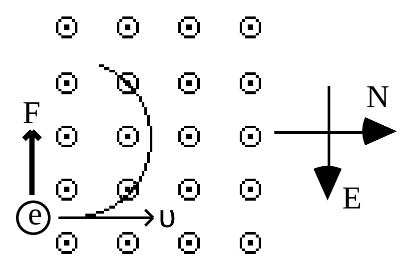
$$\upsilon = \left(\sqrt{\frac{2K}{mc^2}}\right)c = \left(\sqrt{\frac{2(2000eV)}{511x10^3 eV}}\right)(3.00x10^8\ m/s) = 2.65x10^7\ m/s$$

The force on a charged particle in a magnetic field is $\vec{F} = q\vec{\upsilon} \times \vec{B}$.

Since $\vec{\upsilon}$ is perpendicular to $\vec{B}$, $F = (1.60x10^{-19})(2.65x10^7)(0.100) = \underline{\underline{4.24x10^{-13}N}}$.

The force is westward by the right hand rule.

(b)The force on a charged particle in a magnetic field is always perpendicular to the velocity. This means that the motion must be circular and since no work can be done it must be uniform circular motion.

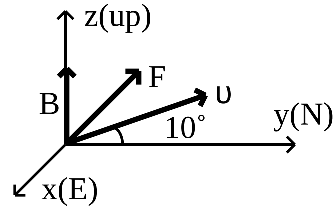


The radius can be found by applying Newton's Second Law,

$$\Sigma\vec{F} = m\vec{a} \qquad q\upsilon B = m\frac{\upsilon^2}{r} \qquad r = \frac{m\upsilon}{qB} = \frac{(9.11x10^{-31})(2.65x10^7)}{(1.60x10^{-19})(0.100)} = \underline{\underline{1.51mm}}$$

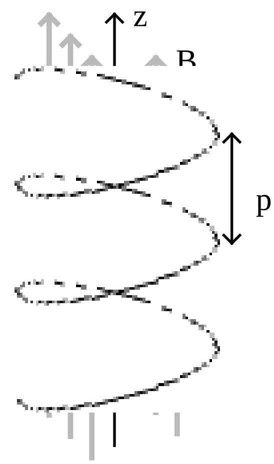
Beyond the Mechanical Universe (vol. 34 Ch 35)

<u>Example 3:</u> Suppose the electron in example 2 was fired at 10.0° above horizontal. Describe the motion.



The force on a charged particle in a magnetic field is $\vec{F} = q\vec{\upsilon}\times\vec{B}$ but now $\vec{\upsilon}$ is not perpendicular to $\vec{B}$ so, $\vec{F} = -e(\upsilon\cos 10^\circ\hat{j} + \upsilon\sin 10^\circ\hat{k})\times B\hat{k} = -e\upsilon B\cos 10^\circ\hat{i}$. Since $\vec{\upsilon}$ is still perpendicular to $\vec{F}$ we will still have uniform circular motion, but the tangential speed will be $\upsilon\cos 10^\circ$. Using the Second Law again,

$$\Sigma\vec{F} = m\vec{a} \qquad q\upsilon_t B = m\frac{\upsilon_t^2}{r} \qquad r = \frac{m\upsilon_t}{qB} = 1.51\text{mm}\ \cos 10^\circ = \underline{\underline{1.49\text{mm}}}.$$

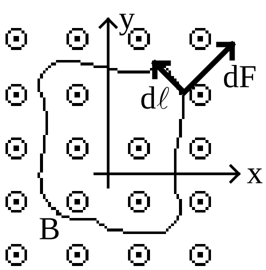


Since there is no force along the z-direction, the z-component of velocity will remain constant. The resulting motion is helical. The "pitch" of the helix can be found from, $p = v_z T = v\sin 10^\circ\ \dfrac{2\pi r}{v\cos 10^\circ} = \underline{\underline{1.65\text{mm}}}$.

Charged particles generally spiral around magnetic field lines.

This phenomena is responsible for the "Northern Lights."

<u>4. Current Loops in a Constant Field</u>

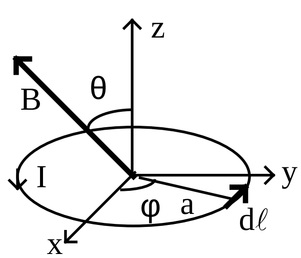


A uniform magnetic field points along the z-direction. An arbitrarily shaped current loop is placed in the field. Let's find the net force on the loop. The force, dF, on a small segment of the loop, dℓ, is given by the definition of magnetic field, $d\vec{F} = Id\vec{\ell}\times\vec{B}$. The total force on the loop is then, $\vec{F} \equiv \int Id\vec{\ell}\times\vec{B}$.

An arbitrary segment is given by $d\vec{\ell} = dx\hat{i} + dy\hat{j} + dz\hat{k}$ and the magnetic field is $\vec{B} = B\hat{k}$.

The cross product is, $d\vec{\ell}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & dy & dz \\ 0 & 0 & B \end{vmatrix} = Bdy\hat{i} - Bdx\hat{j}$. The total force on the loop is

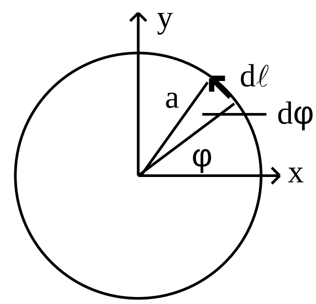
$\vec{F} = \int d\vec{F} = I\left(\oint Bdy\hat{i} - \oint Bdx\hat{j}\right) = IB\left(\hat{i}\oint dy - \hat{j}\oint dx\right) \qquad \vec{F} = 0$ for any shaped loop in a constant magnetic field.



What about the torque on the loop? The book wimps out and does a square loop. We're tough, so let's do a circular loop of radius, a. The loop will be in the x-y plane and B is in the x-z plane and angle θ from the z-axis. This geometry means no loss of generality.

The definition of torque is $\vec{\tau} \equiv \vec{r}\times\vec{F}$, so the torque $d\vec{\tau}$ about the center of the loop on a small segment $d\vec{\ell}$ caused by the force $d\vec{F} = Id\vec{\ell}\times\vec{B}$ must be,

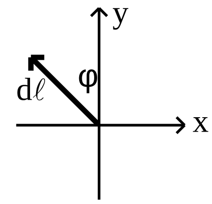
$$d\vec{\tau} = \vec{r}\times d\vec{F} = \vec{r}\times\left(Id\vec{\ell}\times\vec{B}\right).$$



The three vectors we need are $\vec{r} = a\cos\varphi\hat{i} + a\sin\varphi\hat{j}$, $d\vec{\ell} = a\,d\varphi\left(-\sin\varphi\hat{i} + \cos\varphi\hat{j}\right)$ and $\vec{B} = B\left(\sin\theta\hat{i} + \cos\theta\hat{k}\right)$.

$$d\vec{\ell} \times \vec{B} = aB\,d\varphi\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin\varphi & \cos\varphi & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}$$

$$d\vec{\ell} \times \vec{B} = aB\,d\varphi\left(\cos\theta\cos\varphi\hat{i} + \cos\theta\sin\varphi\hat{j} - \sin\theta\cos\varphi\hat{k}\right)$$



$$d\vec{\tau} = \vec{r} \times \left(I\,d\vec{\ell} \times \vec{B}\right) = Ia^2B\,d\varphi\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\varphi & \sin\varphi & 0 \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta\cos\varphi \end{vmatrix}$$

$$d\vec{\tau} = Ia^2B\,d\varphi\left(-\sin\theta\cos\varphi\sin\varphi\hat{i} + \sin\theta\cos^2\varphi\hat{j}\right)$$

$$\vec{\tau} = Ia^2B\sin\theta\left(-\hat{i}\int_0^{2\pi}\cos\varphi\sin\varphi\,d\varphi + \hat{j}\int_0^{2\pi}\cos^2\varphi\,d\varphi\right) = Ia^2B\sin\theta\left(-0\hat{i} + \pi\hat{j}\right) = I\pi a^2B\sin\theta\hat{j}$$

If we make the following definition,

$$\vec{\mu} \equiv I\vec{A} \quad \text{The Defintion of Magnetic Dipole Moment}$$

Then we can write the torque on the dipole as,

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad \text{Torque on a Magnetic Dipole}$$

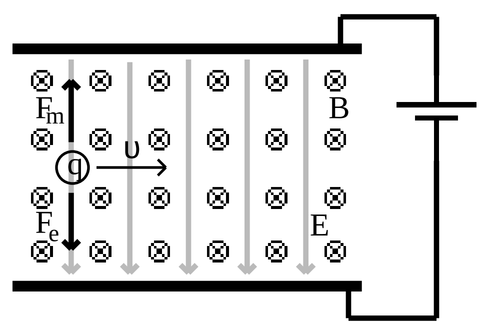
This equation is similar to the result for the torque on an electric dipole. By using this analogy we can write the potential energy of a magnetic dipole in a magnetic field as,

$$U = -\vec{\mu} \cdot \vec{B} \quad \text{Potential Energy of a Magnetic Dipole}$$

Beyond the Mechanical Universe (vol. 34 Ch 14)

5. Magnetic Devices

a) Velocity Selector



Particles of mass m and charge q move at a speed υ into a region with a vertical E-field and a horizontal B-field as shown. It turns out that there will be only one velocity that will allow the particles to be undeflected by the fields. It can be found by using the Second Law, $\Sigma\vec{F} = m\vec{a}$ $\quad F_m - F_e = ma$. Apply the definition of E-field and the magnetic force on a moving charge, $q\upsilon B - qE = ma$. For the undeflected particles,

$$a = 0 \qquad q\upsilon B - qE = 0 \qquad \upsilon = \frac{E}{B}.$$

Example 4: *A mass spectrometer requires charged particles traveling at $2.00\text{x}10^5$m/s. The magnetic field in the device is 0.500T. Find the potential difference across the plates of the velocity selector given their separation is 0.800cm.*

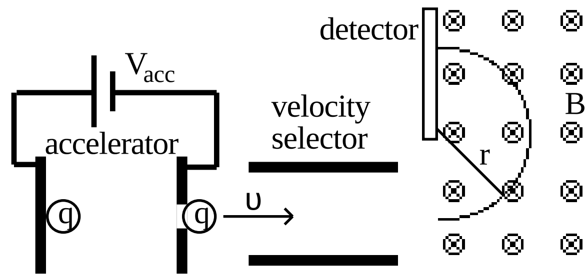
Applying the Second Law, $\Sigma\vec{F} = m\vec{a} \qquad q\upsilon B - qE = 0 \qquad \upsilon = \dfrac{E}{B}$.

Between parallel plates, $E = \dfrac{V}{d}$.

Finally, $\upsilon = \dfrac{E}{B} = \dfrac{V}{Bd} \qquad V = B\upsilon d = (0.500)(2.00\text{x}10^5)(8.00\text{x}10^{-3}) = \underline{\underline{800V}}$

b)Mass Spectrometer

Example 5: *Singley charged chlorine atoms of mass 35 and 37 travel at $2.00\text{x}10^5$m/s as they enter the 0.500T field. Find their separation at the detector.*



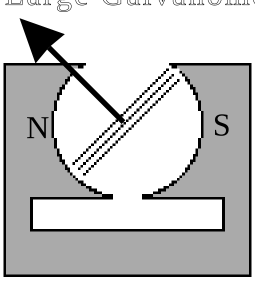
Starting with the Second Law and assuming uniform circular motion,

$\Sigma\vec{F} = m\vec{a} \qquad q\upsilon B = m\dfrac{\upsilon^2}{r}$. Since different masses will have different radii, $r_1 = \dfrac{m_1\upsilon}{qB}$ and $r_2 = \dfrac{m_2\upsilon}{qB}$. The separation at the detector is the difference in the diameters,

$$d = 2(r_2 - r_1) = (m_2 - m_1)\frac{2\upsilon}{qB} = 2(1.67\text{x}10^{-27})\frac{2(2.00\text{x}10^5)}{(1.60\text{x}10^{-19})(0.500)} = \underline{\underline{1.67\text{cm}}}$$

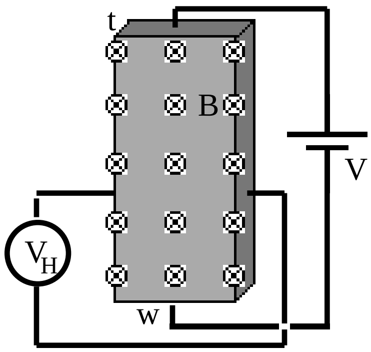
c)Galvanometer

Large Galvanometer



A galvanometer measures current by allowing the current to flow through a coil of wire creating a magnetic dipole. The coil is placed in a magnetic field of the proper shape so that the torque doesn't depend on the angle the loop makes with the horizontal. The torque on a dipole is, $\vec{\tau} = \vec{\mu} \times \vec{B} \qquad \tau = \mu B$. In terms of the magnetic moment of a loop of N coils, $\tau = NIAB$. Notice that this is linear in the current (Twice the current produces twice the torque).

d)Hall Probe



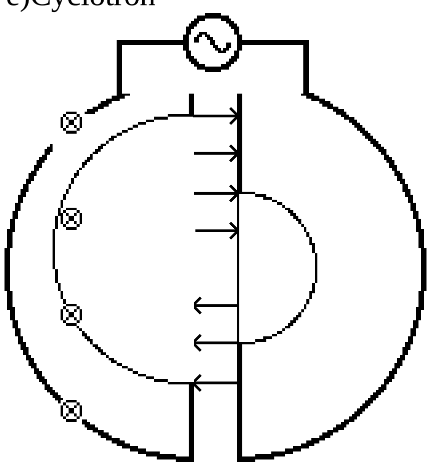
Bring a Hall Probe

The Hall Effect is the first experiment we have encountered that can actually detect the sign of the charge carriers. When a voltage V is applied to the sample, a current I will flow downward (electrons flow upward). If positive charges were flowing downward they would deflect to the right. Negative charges flowing upward would also deflect to the right. So the sign of the charges can be determined by which kind of charge moves to the right.

The Hall Probe shown here can be used to find the strength of a magnetic field. The magnetic force on the electrons that bends them to the right is, $\vec{F} = q\vec{\upsilon} \times \vec{B}$ $\quad F = e\upsilon B$ where υ is the drift velocity which is related to the current density $j = ne\upsilon$ $\quad \frac{I}{wt} = ne\upsilon \quad \upsilon = \frac{I}{newt}$. The force can be thought of as arising from the an effective electric field $F = e\upsilon B \quad eE_{eff} = e\upsilon B \quad E_{eff} = \upsilon B$. This field is related to the "Hall Voltage" $E_{eff} = \upsilon B \quad \frac{V_H}{w} = \upsilon B$. Combing this with the velocity equation and solving for the field,

$$\frac{V_H}{w} = \frac{IB}{newt} \qquad B = (net)\frac{V_H}{I}$$

e)Cyclotron



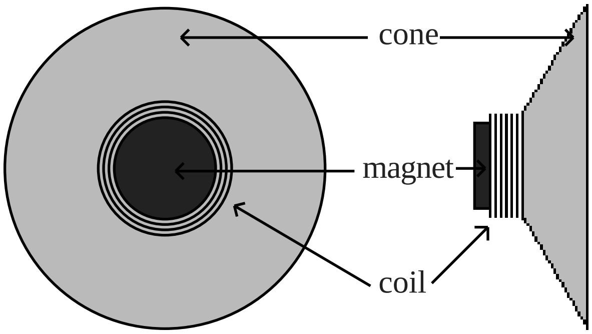
A cyclotron is a system designed to accelerate particles such as a proton to high speeds. It consists of two half cylinders with an alternating potential difference across the gap that separates them. A magnetic field causes the protons to move in circular paths. During the time the proton is inside one of the halves it doesn't accelerate. It is only when it crosses from one side to the other that it feels the potential difference. During the time the proton is inside one of the halves, the sign of the voltage is switched so that when it gets back to the gap it is again accelerated.

Example 6: Find the frequency at which the voltage must alternate in terms of the mass m, the charge q, and the field B.

The frequency must be the reciprocal of the time it takes the particles to complete an orbit because the voltage must go through one complete cycle during this time. This time can be found starting with the Second Law, $\Sigma\vec{F} = m\vec{a} \quad q\upsilon B = m\frac{\upsilon^2}{r} \quad \upsilon = \frac{qBr}{m}$ and using the definition of velocity, $\upsilon = \frac{2\pi r}{T} \quad \frac{2\pi r}{T} = \frac{qBr}{m} \quad f \equiv \frac{1}{T} = \frac{qB}{2\pi m}$. Notice that the radius of orbit cancels out, which makes the power supply easier to construct.

f)Stereo Speaker

Bring a speaker and low frequency amp



A magnet is mounted to the frame of the speaker and a coil of wire is mounted to the cone. The current from the amplifier is sent through the coil. When the current is one direction the force on the coil is one direction. When the current reverses, the force on the coil reverses. The resulting oscillatory motion of the cone creates the sound waves.

Chapter 29 - Summary

The Force Between Current Carrying Wires $F_m = \frac{\mu_o}{2\pi}\frac{I_1 I_2}{r}\ell$

The Definition of the Magnetic Field $\vec{F} \equiv I\vec{\ell} \times \vec{B}$

The Magnetic Force on a Charged Particle $\vec{F} = q\vec{\upsilon} \times \vec{B}$

The Definition of Magnetic Dipole Moment $\vec{\mu} \equiv I\vec{A}$

Torque on a Magnetic Dipole $\vec{\tau} = \vec{\mu} \times \vec{B}$

Potential Energy of a Magnetic Dipole $U = -\vec{\mu} \cdot \vec{B}$