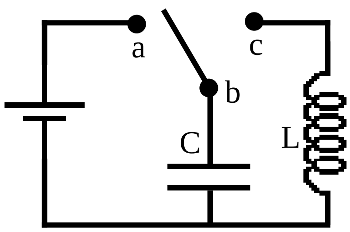
Chapter 33 - Electromagnetic Oscillations and Alternating Current

Problem Set #12 - due:
Ch 33 - 3, 10, 17, 18, 31, 34, 39, 42, 56, 59, 61, 77, 80, 84, 85, 87

Lecture Outline
1. The LC Circuit
2. The LRC Circuit
3. Basic AC Circuits
4. Frequency Filtering Circuits
5. The RLC Tuning Circuit
6. Power in AC Circuits
7. Transformers

We will use the loop and junction theorems to analyze the behavior of circuits that exhibit oscillatory behavior. We will start by allowing the circuits to oscillate at their "natural" frequency, then we will examine the effect of forcing them to oscillate at other frequencies.

1. The LC Circuit



In the circuit at the left, the capacitor is charged when the switch is connected from a to b. Then the switch is connected from b to c. The capacitor begins to lose its charge as it attempts to get current through the inductor. The inductor begins to build up a magnetic field. This field peaks as the charge on the capacitor goes to zero. Then the energy in the inductor begins to create current to charge the capacitor back up. All of this is explained by the loop theorem which requires that the potential difference across the capacitor always equal the potential difference across the inductor,

$$V_c = V_L \qquad \frac{Q}{C} = -L\frac{dI}{dt} \qquad \frac{Q}{C} = -L\frac{d^2Q}{dt^2} \qquad \frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$$

This differential equation is the SHM equation, $\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x \qquad x = A\cos(\omega t + \delta)$.

By analogy,

$$\boxed{Q = Q_m \cos(\omega t + \delta) \text{ where } \omega = \frac{1}{\sqrt{LC}} \qquad \text{The LC Circuit}}$$

Example 1: Show that the above expression for the charge on the capacitor satisfies the loop theorem.

$$Q = Q_m \cos(\omega t + \delta) \qquad \frac{dQ}{dt} = -Q_m \omega \sin(\omega t + \delta) \qquad \frac{d^2Q}{dt^2} = -Q_m \omega^2 \cos(\omega t + \delta) = -\omega^2 Q$$

Substituting into the loop theorem, $-\omega^2 \not{Q} = -\frac{1}{LC}\not{Q} \qquad \omega = \frac{1}{\sqrt{LC}}$.

<u>Example 2:</u> For an LC circuit find (a)the peak current in terms of the peak charge on the capacitor and (b)the fraction of the period between the time the capacitor's charge peaks and the current peaks.

(a)From the definition of current, $I \equiv \frac{dQ}{dt} = -Q_m \omega \sin(\omega t + \delta)$ $\quad \underline{\underline{I_m = Q_m \omega}}$.

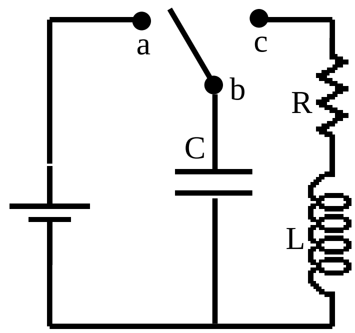
(b)First let's define Q=Q_m when t=0. This establishes the value of δ,

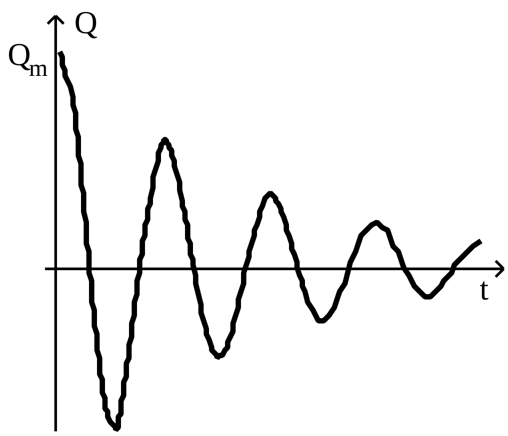
$Q = Q_m \cos(\omega t + \delta) \quad Q_m = Q_m \cos(\delta) \quad \cos(\delta) = 1 \quad \delta = 0$

Note that $I = I_m$ when $\sin(\omega t + \delta) = 1 \quad \omega t + \delta = \frac{\pi}{2} \quad \omega t = \frac{\pi}{2} \quad t = \frac{\pi}{2\omega}$.

Recall, $\omega = \frac{2\pi}{T} \quad t = \frac{\pi}{2\frac{2\pi}{T}} \quad \underline{\underline{t = \tfrac{1}{4}T}}$.

2. The LRC Circuit





The capacitor is charged by connecting a to b. Then the switch is moved from b to c. The oscillations of the LC circuit are damped out by the heat created in the resistor. As usual, the loop theorem gives the details,

$V_L + V_R + V_C = 0 \qquad L\frac{dI}{dt} + IR + \frac{Q}{C} = 0$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

If we assume that R is fairly small then we would expect damped oscillations of the form, $Q = Q_m e^{-\gamma t}\cos\omega_d t$ where Q_m is the initial charge on the capacitor, α is the damping constant, and ω_d is the frequency of oscillation. A graph of such a curve is shown at the left. To check to see if this guess solves the differential equation from the loop theorem, we need the derivatives, $\frac{dQ}{dt} = -Q_m e^{-\gamma t}\{\omega_d \sin\omega_d t + \gamma\cos\omega_d t\}$

and $\frac{d^2Q}{dt^2} = -Q_m e^{-\gamma t}\left\{\left(\omega_d^2 - \gamma^2\right)\cos\omega_d t - 2\gamma\omega_d \sin\omega_d t\right\}$.

Substituting into the loop theorem,

$$-Q_m e^{-\gamma t}\left[L\left\{\left(\omega_d^2 - \gamma^2\right)\cos\omega_d t - 2\gamma\omega_d\sin\omega_d t\right\} + R\{\omega_d\sin\omega_d t + \gamma\cos\omega_d t\} - \frac{1}{C}\cos\omega_d t\right] = 0.$$

Sorting things out, $\left\{\left(\omega_d^2 - \gamma^2\right)L + \gamma R - \frac{1}{C}\right\}\cos\omega_d t + \{\omega_d R - 2\gamma\omega_d L\}\sin\omega_d t = 0$.

If this is to be zero for all values of t, then the coefficients of the sine and cosine must be zero independently.

The sine coefficient gives, $\omega_d R - 2\gamma\omega_d L = 0 \qquad \gamma = \frac{R}{2L}$.

The cosine coefficient requires,

$$\left(\omega_d^2-\gamma^2\right)L+\gamma R-\frac{1}{C}=0 \qquad \left(\omega_d^2-\gamma^2\right)+\gamma\frac{R}{L}-\frac{1}{LC}=0 \qquad \omega_d^2-\gamma^2+2\gamma^2-\frac{1}{LC}=0.$$

Solving for ω_d, $\omega_d^2=\frac{1}{LC}-\gamma^2=\frac{1}{LC}-\left(\frac{R}{2L}\right)^2 \qquad \omega_d=\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}$.

$$Q=Q_m e^{-\frac{Rt}{2L}}\cos\omega_d t \quad \text{where} \quad \omega_d=\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2} \qquad \text{The LRC Circuit}$$

Example 3: *Find the maximum allowed resistance for oscillations to occur.*

Oscillations will occur as long as ω_d is real, so the critical resistance is when,

$$\frac{1}{LC}=\left(\frac{R_c}{2L}\right)^2 \qquad R_c=\sqrt{4\frac{L}{C}}.$$

Example 4: *Find the fraction of energy lost per oscillation for an LRC circuit with small R.*

At some initial time t the charge is, $Q_o=Q_m e^{-\frac{R}{2L}t}\cos\omega_d t$. We want to choose this time so that the charge on the capacitor is peaking so, $t\approx n\frac{2\pi}{\omega_d}$ where n is an integer.

The initial charge is, $Q_o=Q_m e^{-\frac{R}{2L}\left(n\frac{2\pi}{\omega_d}\right)}$.

The energy stored in the capacitor at this time is, $U_o=\frac{1}{2}\frac{Q_o^2}{C}=\frac{1}{2}\frac{Q_m^2}{C}e^{-\frac{R}{L}\left(n\frac{2\pi}{\omega_d}\right)}$.

After one oscillation, $t\approx(n+1)\frac{2\pi}{\omega_d}$, $Q=Q_m e^{-\frac{R}{2L}\left((n+1)\frac{2\pi}{\omega_d}\right)}$, and $U=\frac{1}{2}\frac{Q_m^2}{C}e^{-\frac{R}{L}\left((n+1)\frac{2\pi}{\omega_d}\right)}$.

The fraction of energy lost in one oscillation is,

$$\frac{\Delta U}{U_o}=\frac{U_o-U}{U_o}=\frac{\frac{1}{2}\frac{Q_m^2}{C}e^{-\frac{R}{L}\left(n\frac{2\pi}{\omega_d}\right)}-\frac{1}{2}\frac{Q_m^2}{C}e^{-\frac{R}{L}\left((n+1)\frac{2\pi}{\omega_d}\right)}}{\frac{1}{2}\frac{Q_m^2}{C}e^{-\frac{R}{L}\left(n\frac{2\pi}{\omega_d}\right)}}=1-e^{-\frac{R}{L}\left(\frac{2\pi}{\omega_d}\right)}.$$

For small R use the Taylor expansion, $e^{-x}=1-x+\frac{x^2}{2}-\cdots \qquad \frac{\Delta U}{U_o}\approx 1-\left(1-\frac{2\pi R}{\omega_d L}\right)=\frac{2\pi R}{\omega_d L}$.

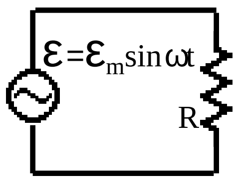
Now we define the "Q-factor" or "Quality" of the circuit as $\frac{\Delta U}{U_o}\equiv\frac{2\pi}{Q} \qquad Q=\frac{\omega_d L}{R}$.

Note: high Q means low loss and low Q means high loss.

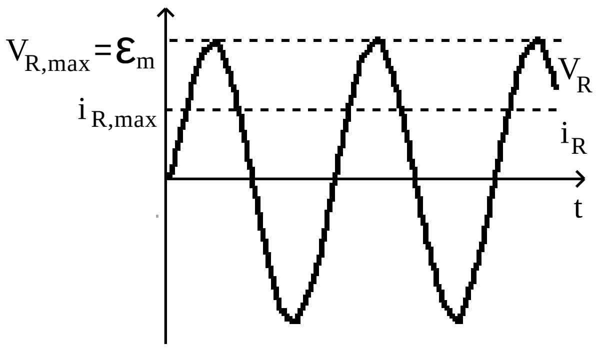
3. Basic AC Cicuits

Instead of allowing circuits to oscillate at the frequency they choose, we can force them to oscillate at any frequency we choose by using an AC power supply. The response of the circuit depends on whether it contains resistors, capacitors, or inductors.

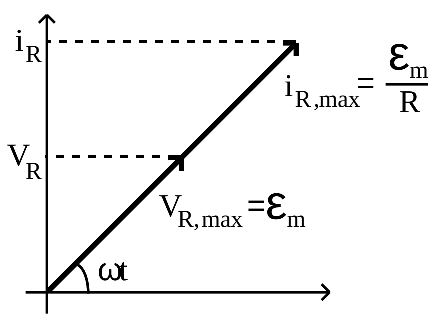
Resistors



The circuit at the left shows a sinusoidally varying power supply connected to a resistor. The resulting current in the circuit is given by Ohm's Rule, $V_R = i_R R \qquad i_R = \frac{V_R}{R}$.



By the loop theorem, the voltage on the resistor must always equal the voltage from the power supply so, $i_R = \frac{\varepsilon_m \sin\omega t}{R} = \frac{\varepsilon_m}{R}\sin\omega t$. The maximum current through the resistor is, $i_{R,max} = \frac{\varepsilon_m}{R}$. Note that both the current and the voltage vary with time as the $\sin\omega t$. We say they are "in phase." This is shown in the graph at the left.



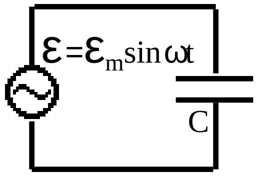
The sketch at the left is called a "phasor diagram." Vectors representing the maximum voltage and maximum current rotate counter-clockwise and make at angle ωt with the positive x-axis. The vertical component of each vector represents the instantaneous value of voltage or current. The ratio of the maximum voltage to the maximum current is defined to be the "impedance."

$$X \equiv \frac{\varepsilon_m}{I_m} \quad \text{The Definition of Impedance}$$

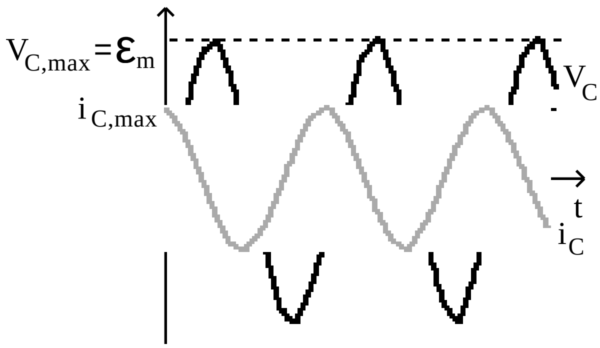
For a resistor, $X \equiv \frac{\varepsilon_m}{I_m} \qquad X_R = \frac{\varepsilon_m}{\frac{\varepsilon_m}{R}} = R$.

$$X_R = R \quad \text{The Impedance of a Resistor}$$

Capacitors



The circuit at the left shows a sinusoidally varying power supply connected to a capacitor. The resulting current in the circuit can be found from the definition of capacitance,

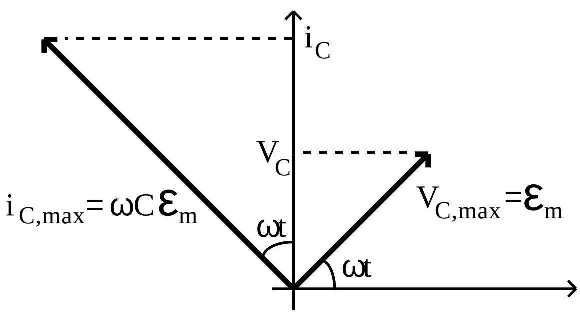


$Q_C = CV_C$ and the definition of current $i_C = \frac{dQ_C}{dt}$.

By the loop theorem, the voltage on the capacitor must always equal the voltage from the power supply so,

$$i_C = \frac{d}{dt}\left(CV_C\right) = C\frac{d}{dt}\left(\mathcal{E}_m \sin\omega t\right) = \omega C\mathcal{E}_m \cos\omega t .$$

The maximum current through the capacitor is, $i_{C,max} = \omega C\mathcal{E}_m$. Note that the current and the voltage are out of phase. The current peaks at an earlier time than the voltage. This is shown in the graph at the left.



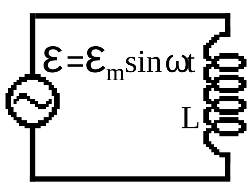
The phasor diagram for this circuit is shown at the left. The current leads the voltage by 90˚.

The ratio of the maximum voltage to the maximum current is defined to be the impedance,

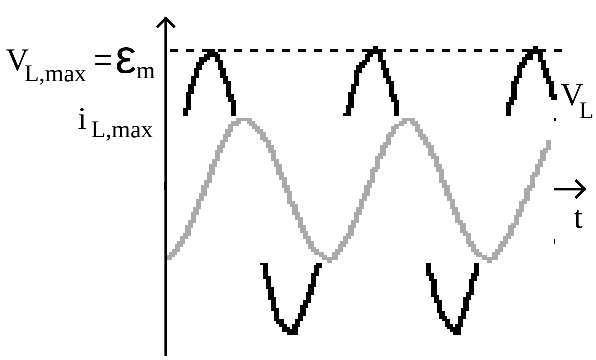
$$X \equiv \frac{\mathcal{E}_m}{I_m} \qquad X_C = \frac{\mathcal{E}_m}{\omega C\mathcal{E}_m} = \frac{1}{\omega C} .$$

$X_C = \frac{1}{\omega C}$ The Impedance of a Capacitor

Inductors



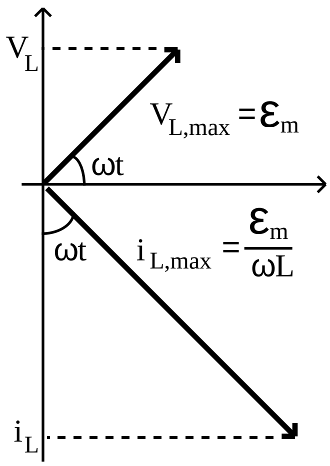
The circuit at the left shows a sinusoidally varying power supply connected to an inductor. The resulting current in the circuit can be found from the definition of self-inductance $V_L = L\frac{di_L}{dt}$. By the loop theorem, the voltage on the inductor must always equal the voltage from the power supply so,



$$\mathcal{E}_m \sin\omega t = L\frac{di_L}{dt} \qquad i_L = -\frac{\mathcal{E}_m}{\omega L}\cos\omega t .$$

The maximum current through the inductor is,

$i_{L,max} = \frac{\mathcal{E}_m}{\omega L}$. Note that the current and the voltage are again out of phase. The current peaks at a later time than the voltage. This is shown in the graph at the left.



The phasor diagram for this circuit is also shown at the left. The current lags the voltage by 90˚. The ratio of the maximum voltage to the maximum current is defined to be the impedance, $X \equiv \frac{\mathcal{E}_m}{I_m}$ $\quad X_L = \frac{\mathcal{E}_m}{\frac{\mathcal{E}_m}{\omega L}} = \omega L$.

$X_L = \omega L$ The Impedance of an Inductor

Now that we see that the impedance of inductors and capacitors depends on the frequency of the power source we can begin to design circuits that have a frequency dependence such as crossovers and tuners. The concept of phasor diagrams cam be extended to more complex circuits.

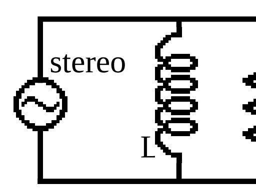
4. Frequency Filtering Circuits

$X_L = \omega L$ Inductors stop high frequency and pass low frequency.

$X_C = \frac{1}{\omega C}$ Capacitors stop low frequency and pass high frequency.

$X_R = R$ Resistors don't care about frequency.

Example 5: A stereo signal contains frequencies from 20.0Hz to 20.0kHz. A tweeter has a resistance of 8.00Ω and functions well between 5.00kHz and 20.0kHz. (a)Design a circuit which will use an inductor to filter out the low frequencies. (b)Find the inductance so that it has the same peak current as the tweeter at 5.00kHz. (c)Find the ratio of the peak currents at 10.0kHz, (d)at 1.00kHz. (e)Include the woofer in the circuit.



(a)The power supply will send a variety of frequencies toward the speaker. Putting an inductor across the speaker allows the low frequencies to bypass the tweeter.

(b)The voltage across the inductor must always equal the voltage across the tweeter according to the loop theorem so, $\mathcal{E}_m = i_{L,max} X_L = i_{R,max} X_R$.

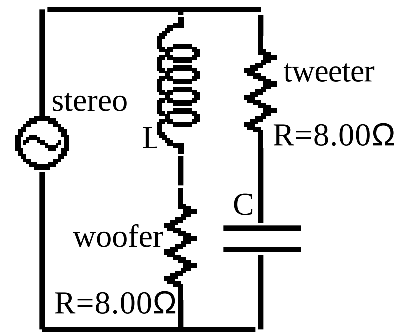
At 5.00kHz the peak current is the same in both the resistor and the inductor so, $i_{L,max} = i_{R,max}$ which means that $X_L = X_R \quad \omega L = R \quad L = \frac{R}{\omega} = \frac{R}{2\pi f} = \frac{8.00}{2\pi(5000)} = \underline{\underline{0.255\text{mH}}}$.

(c)It's still true that

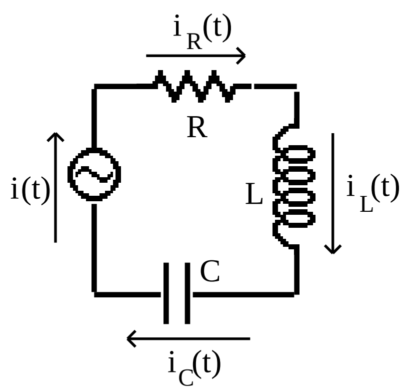
$$\mathcal{E}_m = i_{L,max} X_L = i_{R,max} X_R \qquad \frac{i_{R,max}}{i_{L,max}} = \frac{X_L}{X_R} = \frac{2\pi f L}{R} = \frac{2\pi(10)(0.255)}{8.00} = \underline{\underline{2.00}}.$$

(d)Again, $\mathcal{E}_m = i_{L,max} X_L = i_{R,max} X_R \qquad \frac{i_{R,max}}{i_{L,max}} = \frac{X_L}{X_R} = \frac{2\pi f L}{R} = \frac{2\pi(1)(0.255)}{8.00} = \underline{\underline{0.200}}$.

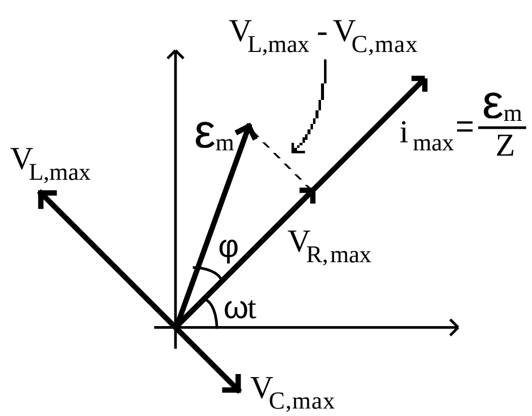
(e)The woofer should go in series with the inductor since the low frequencies prefer this leg of the circuit. Usually a capacitor is put in the leg with the tweeter to adjust the frequency that produces equal currents. This type of circuit is called a "crossover."



5. The RLC Tuning Circuit



In the RLC series circuit, the inductor stops the high frequencies and the capacitor stops the low frequencies. Only frequencies in between can make it through the circuit. Let's find out the relationship between the peak current and the frequency. The junction theorem requires the current at any time to be equal in all the circuit elements, $i(t) = i_R(t) = i_L(t) = i_C(t)$. The loop theorem demands that the sum of the voltage drops at any time add up to the supply voltage, $\varepsilon(t) = V_R(t) + V_L(t) + V_C(t)$.



Since the voltages are out of phase we need to add up the phasors. According to the phasor diagram,

$$\varepsilon_m = \sqrt{V_{R,max}^2 + \left(V_{L,max} - V_{C,max}\right)^2}$$

Using the definition of impedance,

$$\varepsilon_m = \sqrt{\left(i_{max}R\right)^2 + \left(i_{max}X_L - i_{max}X_C\right)^2}\,.$$

Factoring out the peak current,

$$\varepsilon_m = i_{max}\sqrt{R^2 + \left(X_L - X_C\right)^2}\,.$$

Using the definition of impedance, $X \equiv \frac{\varepsilon_m}{I_m} = \sqrt{R^2 + \left(X_L - X_C\right)^2}$.

Usually, the symbol Z is used for the total impedance of a circuit.

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} \quad \text{RLC Series Circuit: Impedance}$$

The phase angle between the voltage and current can be found from the definition of tangent,

$$\tan\varphi = \frac{V_{L,max} - V_{C,max}}{V_{R,max}} = \frac{i_{max}X_L - i_{max}X_C}{i_{max}R} = \frac{X_L - X_C}{R}$$

$$\tan\varphi = \frac{X_L - X_C}{R} \quad \text{RLC Series Circuit: Phase Angle}$$

Example 6: An RLC series circuit contains a 20.0Ω resistor, a 180mH inductor and a 60.0μF capacitor connected to a 110V-rms 60.0Hz power supply. Find the rms current and voltage for each circuit element.

Since all the circuit elements are in series they have the same current. To find this current we need the total impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$. The impedance for each element is, R=20.0Ω,

$X_L = \omega L = 2\pi f L = 2\pi(60.0)(0.180) = 67.9\Omega$ and

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0)(60.0\text{x}10^{-6})} = 44.2\Omega.$$

The total impedance is,

$$Z = \sqrt{(20.0)^2 + (67.9 - 44.2)^2} = 31.0\Omega.$$

The current can be found by the definition of impedance,

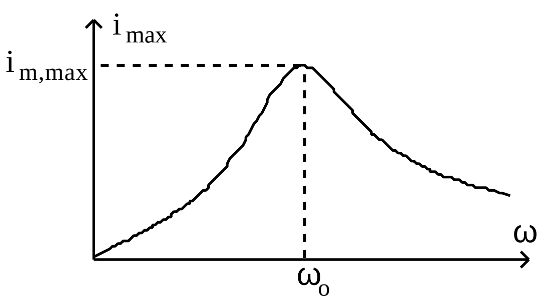
$$X \equiv \frac{\mathcal{E}_m}{I_m} = \frac{\mathcal{E}_m/\sqrt{2}}{I_m/\sqrt{2}} = \frac{\mathcal{E}_{rms}}{I_{rms}} \qquad I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{110}{31.0} = \underline{\underline{3.55A}}.$$

The rms voltage on each one can be found from their impedances:

$V_{R,rms} = i_{rms}R = \underline{\underline{71.0V}}$, $V_{L,rms} = i_{rms}X_L = \underline{\underline{241V}}$ and $V_{C,rms} = i_{rms}X_C = \underline{\underline{157V}}$.

Why don't these add up to the supply voltage? These voltages don't happen at the same time.

How can they be greater than the supply voltage? The other voltages are negative at these times.

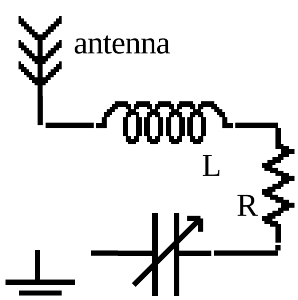


The current that passes through the circuit depends on the frequency. This is called the "frequency response."

$$\mathcal{E}_m = i_{max}\sqrt{R^2 + (X_L - X_C)^2} \qquad i_{max} = \frac{\mathcal{E}_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

The peak current is plotted against the frequency at the left. Note that the capacitor cuts off the low frequencies and the inductor cuts off the high frequencies. The frequency for maximum current is called the "resonance frequency."

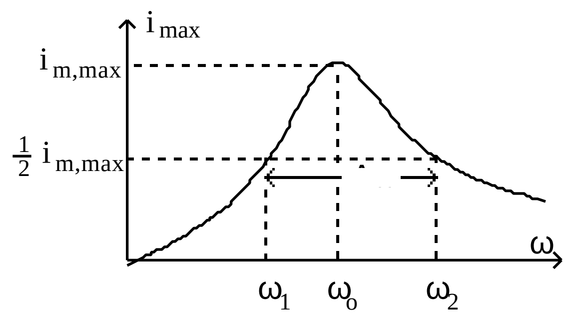
This occurs when $\omega L - \frac{1}{\omega C} = 0 \qquad \omega_o = \frac{1}{\sqrt{LC}}$



$$\omega_o = \frac{1}{\sqrt{LC}}$$ RLC Series Circuit: Resonance Frequency

The fact that only a small range of frequencies make it through this circuit make it ideal for tuning as in a radio. A typical tuning circuit is shown at the left. The tuning is usually accomplished by varying the capacitance.

Example 7: Find the full width of the resonance curve when the peak current is half its maximum value.



$$i_{max} = \tfrac{1}{2} i_{m,max}$$

Using the definition of impedance, the total impedance of the RLC circuit and the fact that the term in parentheses is zero at resonance,

$$\frac{\varepsilon_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{1}{2}\frac{\varepsilon_m}{\sqrt{R^2 + \left(\omega_o L - \frac{1}{\omega_o C}\right)^2}} \qquad \frac{\varepsilon_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{\varepsilon_m}{2R}$$

Canceling the driving voltage and squaring both sides,

$$R^2 + \left(\omega L - \tfrac{1}{\omega C}\right)^2 = 4R^2 \qquad \left(\omega L - \tfrac{1}{\omega C}\right)^2 = 3R^2 \qquad \omega L - \tfrac{1}{\omega C} = \sqrt{3}R\,.$$

Assume the frequencies we want, ω, are symmetrically located about the resonance frequency,

$$\omega = \omega_o \pm \frac{\Delta\omega}{2} = \omega_o\left(1 \pm \frac{\Delta\omega}{2\omega_o}\right) \qquad \frac{1}{\omega} = \frac{1}{\omega_o}\left(1 \pm \frac{\Delta\omega}{2\omega_o}\right)^{-1} \approx \frac{1}{\omega_o}\left(1 \mp \frac{\Delta\omega}{2\omega_o}\right)$$

Substituting back into the previous equation,

$$\left(1 \pm \frac{\Delta\omega}{2\omega_o}\right)\omega_o L - \left(1 \mp \frac{\Delta\omega}{2\omega_o}\right)\frac{1}{\omega_o C} = \sqrt{3}R \qquad \left(\omega_o L - \frac{1}{\omega_o C}\right) \pm \frac{\Delta\omega}{2\omega_o}\left(\omega_o L + \frac{1}{\omega_o C}\right) = \sqrt{3}R$$

Since $\omega_o = \dfrac{1}{\sqrt{LC}}$ $\quad \omega_o L = \frac{1}{\omega_o C}$, we get, $0 \pm \dfrac{\Delta\omega}{2\omega_o}\left(2\omega_o L\right) = \sqrt{3}R \qquad \Delta\omega = \pm\dfrac{\sqrt{3}R}{L}$.

The width of the curve only depends on R and L not on C. Radios use capacitors to tune so that the stations can be equally spaced.

$$\Delta\omega = \frac{\sqrt{3}R}{L} \quad \text{RLC Series Circuit: Full Width Half Maximum}$$

Example 8: Design an RLC series circuit that is an FM tuner. The frequency of a typical FM station is 100MHz. The spacing between stations is 0.200MHz. Assume the circuit requires a 5.00mA peak current and the antenna induces a 100mV emf.

The resistance can be found from the relationship between the peak current and peak voltage,

$$i_{max} = \frac{\varepsilon_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \qquad i_{m,max} = \frac{\varepsilon_m}{R} \qquad R = \frac{\varepsilon_m}{i_{m,max}} = \frac{100\text{mV}}{5.00\text{mA}} = \underline{\underline{20.0\Omega}}\,.$$

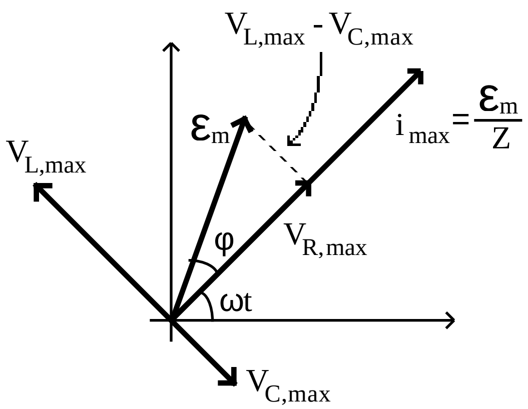
The inductance is fixed by the spacing between stations,

$$\Delta\omega = \frac{\sqrt{3}R}{L} \qquad L = \frac{\sqrt{3}R}{\Delta\omega} = \frac{\sqrt{3}(20.0)}{2\pi(0.200\text{x}10^6)} = \underline{\underline{27.6\mu\text{H}}}.$$

The capacitance is used to tune for the desired frequency,

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad C = \frac{1}{4\pi^2 f_o^2 L} = \frac{1}{4\pi^2(1.00\text{x}10^8)^2(27.6\text{x}10^{-6})} = \underline{\underline{0.0918\text{pF}}}$$

6. Power in AC Circuits



Energy supplied to capacitors is stored in the electric field between the plates. This energy will be returned in full when the capacitor discharges. Energy supplied to inductors is stored in the magnetic field of the coils. This energy is also returned in full when the field collapses. The only place where energy is "lost" to the circuit is in the resistor where heat is produced and not returned. The power consumed by a resistor is $P = iV_R$ as before. The only difference in an AC circuit is that the current and voltage oscillate with time, $P(t) = (i_{max}\sin\omega t)(V_{R,max}\sin\omega t) = i_{max}V_{R,max}\sin^2\omega t$. So the peak power consumed is, $P = i_{max}V_{R,max}$. In terms of the peak voltage supplied to the circuit, $P = i_{max}\mathcal{E}_m\cos\varphi$. The $\cos\varphi$ is called the "power factor." Using Ohm's Rule, $P = i_{max}V_{R,max} = i_{max}(i_{max}R) = i_{max}^2R$ as you might suspect. The average power consumed is, $P_{av} = \overline{i^2}R = i_{rms}^2R$.

$$P_{av} = i_{rms}^2R \quad \text{Average Power Consumed}$$

<u>Example 9:</u> For the circuit of example 2, find the average power delivered (a)at 60Hz and (b)at the resonance frequency.

(a)The average power delivered will match the average power consumed. Using the numbers from example 2,

$$P_{av} = i_{rms}^2R = (3.55)^2(20.0) = \underline{\underline{252W}}.$$

(b)At resonance, the impedance is equal to the resistance, so the rms current is,

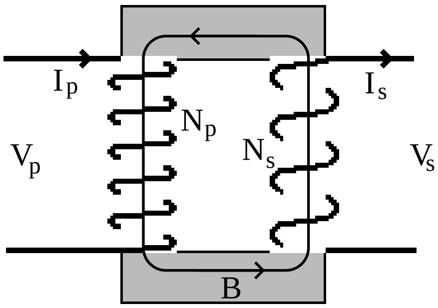
$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{\mathcal{E}_{rms}}{R} = \frac{110}{20.0} = 5.50A$$

The average power consumed will be,

$$P_{av} = i_{rms}^2R = (5.50)^2(20.0) = \underline{\underline{605W}}.$$

Since the current is a maximum at resonance, the power consumed also is maximum.

7. Transformers



Transformers are used to convert current to voltage or vice-versa. Assuming no losses to heat the Law of Conservation of Energy requires that the power that comes in must equal the power that goes out, $I_pV_p = I_sV_s$. The changing current in the primary coils creates a changing magnetic field in the iron. This changing magnetic field through the secondary coils creates an induced voltage according to Faraday's Law.

Assuming that the iron keeps the flux the same in both coils, $\Phi_p = \Phi_s \quad \frac{d\Phi_p}{dt} = \frac{d\Phi_s}{dt}$.

Using Faraday's Law, $\frac{d\Phi_p}{dt} = \frac{d\Phi_s}{dt} \quad \frac{V_p}{N_p} = \frac{V_s}{N_s}$. In summary,

$I_pV_p = I_sV_s$	Transformers: Conservation of Energy
$\frac{V_p}{N_p} = \frac{V_s}{N_s}$	Equal Flux

If the voltage increases we call it a "step-up" transformer, if it decreases we call it a "step-down" transformer.

> *Example 10: Design a transformer to convert standard 110V to 12V for a laptop power supply. The secondary side has 100 turns and delivers 100mA. Find the number of turns needed in the primary and the current the primary will draw.*
>
> The requirement of equal flux through the primary and secondary yields,
>
> $$\frac{V_p}{N_p} = \frac{V_s}{N_s} \qquad N_p = N_s\frac{V_p}{V_s} = 100\frac{110}{12} = \underline{\underline{917\text{turns}}}.$$
>
> The Law of Conservation of Energy requires,
>
> $$I_pV_p = I_sV_s \qquad I_p = I_s\frac{V_s}{V_p} = (100)\frac{12}{110} = \underline{\underline{10.9\text{mA}}}.$$

Chapter 33 - Summary

The LC Circuit $Q = Q_m \cos(\omega t + \delta)$ where $\omega = \frac{1}{\sqrt{LC}}$

The LRC Circuit $Q = Q_m e^{-\frac{Rt}{2L}} \cos \omega_d t$ where $\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

The Definition of Impedance $X \equiv \frac{\varepsilon_m}{I_m}$

The Impedance of a Resistor $X_R = R$

The Impedance of a Capacitor $X_C = \frac{1}{\omega C}$

The Impedance of an Inductor $X_L = \omega L$

RLC Series Circuit:

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Phase Angle $\tan \varphi = \frac{X_L - X_C}{R}$

Resonance Frequency $\omega_o = \frac{1}{\sqrt{LC}}$

Full Width Half Maximum $\Delta\omega = \frac{\sqrt{3}R}{L}$

Average Power Consumed $P_{av} = i_{rms}^2 R$

Transformers

Conservation of Energy $I_p V_p = I_s V_s$

Equal Flux $\frac{V_p}{N_p} = \frac{V_s}{N_s}$