Chapter 34 - Electromagnetic Waves

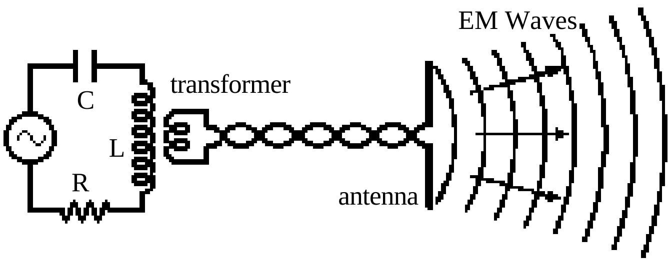
Problem Set #13 - due:
Ch 34 - 2, 6, 8, 12, 16, 17, 20, 25, 28, 35, 45, 47

Since Maxwell's Equations summarize everything we know about electricity and magnetism, they should lead us to an understanding of the properties of electromagnetic waves.

Lecture Outline
1. Producing and Detecting Electromagnetic Waves
2. Properties of Electromagnetic Waves
3. Maxwell's Equations and Waves in Free Space
4. The Electromagnetic Spectrum
5. The Poynting Vector
6. Radiation Pressure and Momentum Transfer

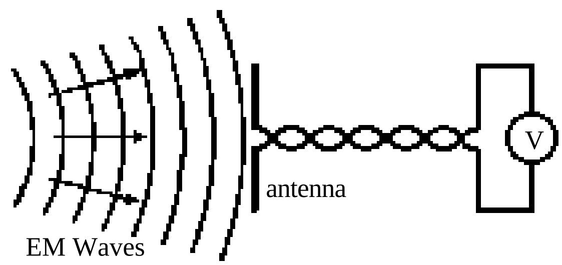
1. Producing and Detecting Electromagnetic Waves

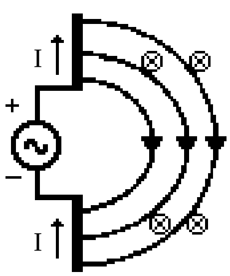
laser and microwave generator & detector



Electromagnetic waves can be produced by an oscillating circuit connected to an antenna as shown at the left. The oscillating charges in the antenna set up electric and magnetic fields.

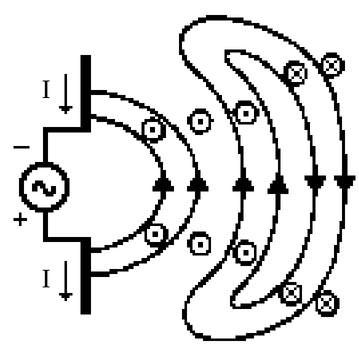
EM waves can be detected with an antenna as shown at the right. The charges in the antenna are forces to oscillate by the EM waves and the resulting voltages can be detected with a voltmeter.

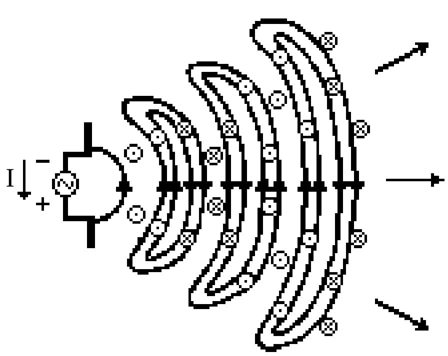




The waves are initially produced by the charges on the antenna. The charge itself produces an E-field and the motion of the charges (current) produces a B-field as shown at the left.

Later when the current switches directions, the fields also switch directions near the antenna. However, the changing fields away from the antenna induce more fields according to Maxwell's Laws of Electricity and Magnetism as shown at the right.



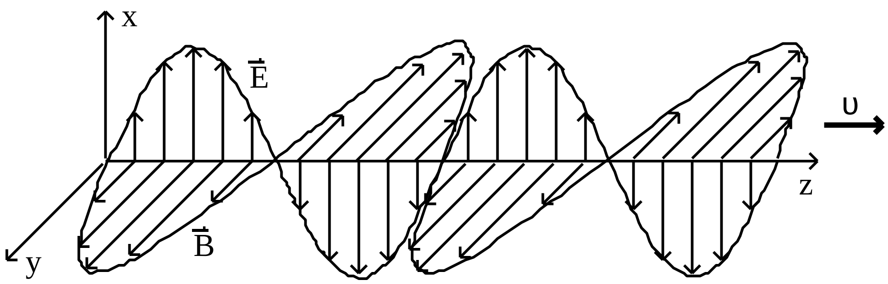


The amazing thing is that these field are self sustaining. The changing magnetic field producing an electric field and the changing electric field produces a magnetic field. This is the nature of the EM waves shown at the left.

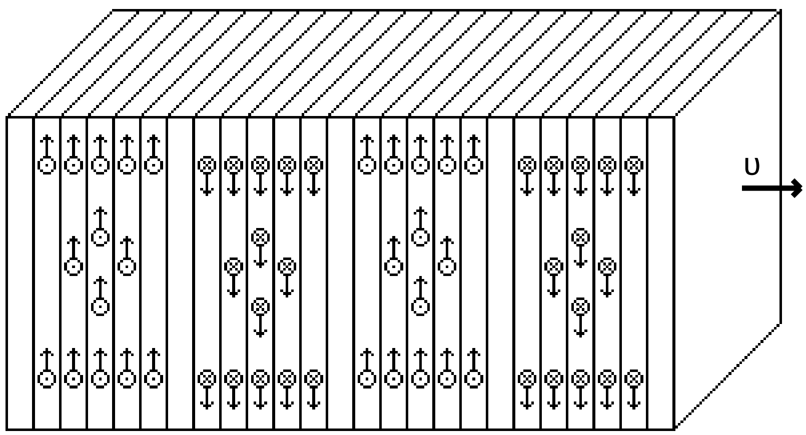
2. Properties of Electromagnetic Waves

- They continue to travel after the source is turned off.
- They travel through empty space.
- They always travel at the same constant speed.
- The electric field is always perpendicular to the magnetic field.
- The velocity is perpendicular to both the electric field and the magnetic field.
- The ratio of the peak electric field to the peak magnetic field equals the speed of the waves.

There are a couple of ways to represent these waves that illustrate these properties.



The electric and magnetic fields oscillate in amplitude in space and they travel to the right as time goes on. This shows the varying strength of the fields, but it doesn't illustrate the fact that these waves are for all practical purposes infinite in the x and y direction.



This illustration indicates the infinite extent in the x and y directions but it is hard to visualize that things are waving. Regardless of how you visualize the waves they must be explained by Maxwell's Equations.

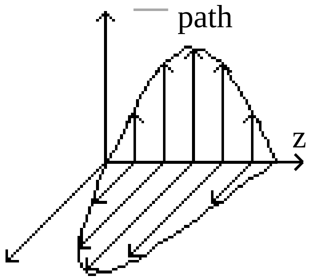
3. Maxwell's Equations and Waves in Free Space

Maxwell's Equations in empty space (free of charges and currents) are:

Gauss's Law for Electricity $\oint \vec{E} \cdot d\vec{A} = 0$ Gauss's Law for Magnetism $\oint \vec{B} \cdot d\vec{A} = 0$

Faraday's Law of Induction $\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi_B}{dt}$ Ampere's Law $\oint \vec{B} \cdot d\vec{s} = \mu_o \varepsilon_o \dfrac{d\Phi_e}{dt}$

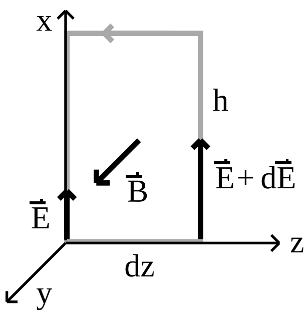
According to Faraday's Law changing magnetic fields make electric fields and according to Ampere's Law changing electric fields make magnetic fields. This is the essence of the propagation of electromagnetic waves. The fields produce each other as they change in space and time. All the properties of EM waves listed above can be explained by applying Maxwell's Equations.



Consider a rectangular path in the x-z plane of height h and width dz as shown.

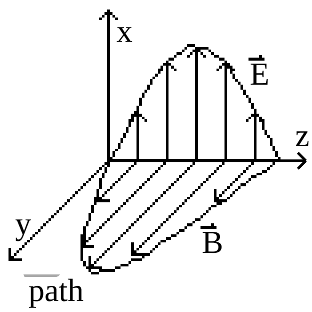
Now apply Faraday's Law, $\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ $\quad \oint \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$.

The path integral only has contributions along vertical portions of the path, $\oint \vec{E}\cdot d\vec{s} = (E + dE)h - Eh = hdE$.



The flux integral can be done because the B-field is nearly constant over the rectangle, $\int \vec{B}\cdot d\vec{A} = Bhdz$.

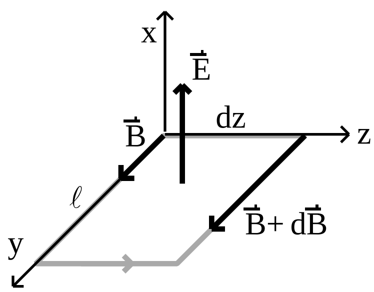
Putting these results into Faraday's Law, $\cancel{h}dE = -\frac{d}{dt}\left(B\cancel{h}dz\right)$. The only thing changing with time is the B-field so, $dE = -dz\frac{dB}{dt}$ $\quad \frac{dE}{dz} = -\frac{dB}{dt}$. If this were done with total mathematical rigor the derivatives are actually partial derivatives. So the relationship between E and B from Faraday's Law is, $\frac{\partial E}{\partial z} = -\frac{\partial B}{\partial t}$.



Now consider a rectangular path in the y-z plane of length ℓ and width dz as shown. Apply Ampere's Law,

$\oint \vec{B}\cdot d\vec{s} = \mu_o\varepsilon_o\frac{d\Phi_e}{dt}$ $\quad \oint \vec{B}\cdot d\vec{s} = \mu_o\varepsilon_o\frac{d}{dt}\int \vec{E}\cdot d\vec{A}$.

The path integral only has contributions along portions parallel to the y-axis, $\oint \vec{B}\cdot d\vec{s} = B\ell - (B + dB)\ell = -\ell dB$.



The flux integral can be done because the E-field is nearly constant over the rectangle, $\int \vec{E}\cdot d\vec{A} = E\ell dz$.

Putting these results into Ampere's Law,

$-\ell dB = \mu_o\varepsilon_o\frac{d}{dt}\left(E\ell dz\right)$ $\quad -\frac{dB}{dz} = \mu_o\varepsilon_o\frac{dE}{dt}$. Because only the E-field is changing with time. Again, these are actually partial derivatives. So the relationship between E and B from Ampere's Law is, $\frac{\partial B}{\partial z} = -\mu_o\varepsilon_o\frac{\partial E}{\partial t}$.

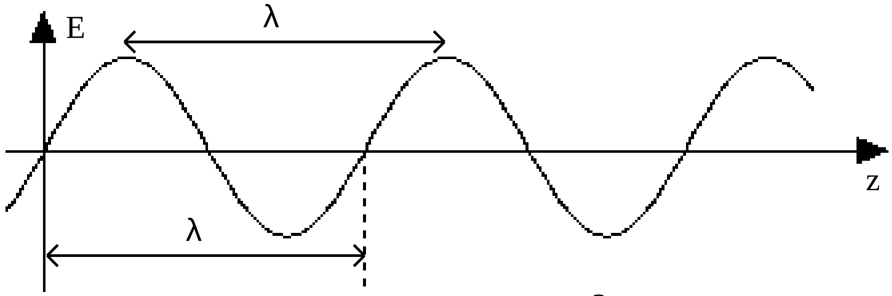
In summary, $\frac{\partial E}{\partial z} = -\frac{\partial B}{\partial t}$ and $\frac{\partial B}{\partial z} = -\mu_o\varepsilon_o\frac{\partial E}{\partial t}$.

Taking the derivative with respect to z of the first equation and the derivative with respect to t of the second equation gives, $\frac{\partial^2 E}{\partial z^2} = -\frac{\partial^2 B}{\partial z\partial t}$ and $\frac{\partial^2 B}{\partial z\partial t} = -\mu_o\varepsilon_o\frac{\partial^2 E}{\partial t^2}$.

Combing the results produces, $\frac{\partial^2 E}{\partial z^2} = \mu_o\varepsilon_o\frac{\partial^2 E}{\partial t^2}$. This is the "Wave Equation." There is a similar wave equation for the magnetic field. So Maxwell's Equations are consistent with the possibility of self sustaining transverse waves.

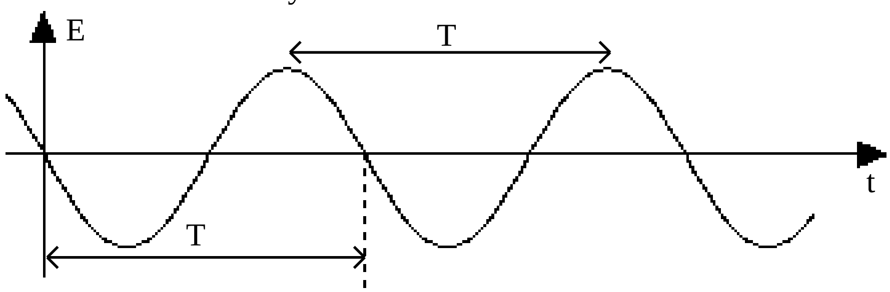
Example 1: Show that $E = E_m \sin(kz - \omega t)$ *is a wave moving in the positive z direction. Express k and* ω *in terms of the wavelength,* λ*, and the frequency, f and find their relationship to the velocity.*

A graph of E vs. z at t=0 is shown. This is like a photograph of the wave.



Notice that z=λ when $kz - \omega t = 2\pi \quad k\lambda - 0 = 2\pi \quad k = \frac{2\pi}{\lambda}$. k is called the "wave number."

Below is a graph of E vs. t at z=0. This is created by standing at one place and measuring E at different times as the wave moves by.



Notice that t=T when $kz - \omega t = 2\pi \quad 0 - \omega T = 2\pi \quad \omega = \frac{2\pi}{T} = 2\pi f$.

Since E is negative after a short time, you can see from the graph of E vs. z that the wave must be moving to the right. If you stand at z=0 and wait for one period, then the wave will have traveled one full wavelength. The speed of the wave can be found using the definition of speed,

$$v \equiv \frac{\Delta z}{\Delta t} = \frac{\lambda}{T} = \lambda f.$$

In terms of ω and k, $v = \lambda f = \frac{2\pi}{k}\frac{\omega}{2\pi} = \frac{\omega}{k}$.

$\boxed{k \equiv \frac{2\pi}{\lambda} \quad \text{The Definition of Wave Number}}$

$\boxed{\frac{\omega}{k} = \lambda f = c \quad \text{The Frequency-Wavelength Relationship}}$

<u>*Example 2:*</u> *Find the conditions under which* $E = E_m \sin(kz - \omega t)$ *is a solution to the wave equation.*

The wave equation is $\dfrac{\partial^2 E}{\partial z^2} = \mu_o \varepsilon_o \dfrac{\partial^2 E}{\partial t^2}$.

Taking the space derivatives, $\dfrac{\partial E}{\partial z} = kE_m \cos(kz - \omega t)$ and $\dfrac{\partial^2 E}{\partial z^2} = -k^2 E_m \sin(kz - \omega t) = -k^2 E$

and the time derivatives, $\dfrac{\partial E}{\partial t} = -\omega E_m \cos(kz - \omega t)$ and $\dfrac{\partial^2 E}{\partial z^2} = -\omega^2 E_m \sin(kz - \omega t) = -\omega^2 E$.

Substituting into the wave equation, $-k^2 E = -\mu_o \varepsilon_o \omega^2 E \quad \dfrac{\omega}{k} = \dfrac{1}{\sqrt{\mu_o \varepsilon_o}}$.

From example 1 we know this ratio must be the speed of the waves,

$v = \dfrac{1}{\sqrt{\mu_o \varepsilon_o}} = 3.00\text{x}10^8\,\text{m/s} \equiv c$, the speed of light. Light is an EM wave!

In order for EM waves to exist, they must travel at this speed regardless of the frequency or wavelength.

$$\boxed{c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \qquad \text{The Speed of EM Waves}}$$

<u>*Example 3:*</u> *Show that any function of kz±ωt is a solution to the wave equation.*

The general form of the wave equation is, $\dfrac{\partial^2 f}{\partial z^2} = \dfrac{1}{\upsilon^2}\dfrac{\partial^2 f}{\partial t^2}$. Define $f(u) \equiv f(kz \pm \omega t)$.

We need the derivatives with respect to z, $\dfrac{\partial f}{\partial z} = \dfrac{\partial f}{\partial u}\dfrac{\partial u}{\partial z} = \dfrac{\partial f}{\partial u}\, k$ and

$$\frac{\partial^2 f}{\partial z^2} = \frac{\partial}{\partial z}\frac{\partial f}{\partial u}\, k = k\frac{\partial u}{\partial z}\frac{\partial^2 f}{\partial u^2} = k^2\frac{\partial^2 f}{\partial u^2}$$

and with respect to t, $\dfrac{\partial f}{\partial t} = \dfrac{\partial f}{\partial u}\dfrac{\partial u}{\partial t} = \dfrac{\partial f}{\partial u}(\pm\omega)$ and

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\frac{\partial f}{\partial u}(\pm\omega) = \pm\omega\frac{\partial u}{\partial t}\frac{\partial^2 f}{\partial u^2} = \omega^2\frac{\partial^2 f}{\partial u^2}.$$

Plugging into the wave equation, $\dfrac{\partial^2 f}{\partial z^2} = \dfrac{1}{\upsilon^2}\dfrac{\partial^2 f}{\partial t^2} \qquad k^2\dfrac{\partial^2 f}{\partial u^2} = \dfrac{1}{c^2}\omega^2\dfrac{\partial^2 f}{\partial u^2} \qquad \underline{\underline{\omega = ck}}$. It works!

If any function of kz±ωt is a solution to the wave equation why do we usually discuss solutions of the form $E = E_m \sin(kz - \omega t)$ and $B = B_m \sin(kz - \omega t)$?

Fourier's Theorem states $f(kz - \omega t) = \sum_i a_i \sin(k_i z - \omega_i t)$.

Any function can be written as a linear combination of sine and cosine waves.

<u>*Example 4:*</u> *Find the ratio of the peak electric field to the peak magnetic field.*

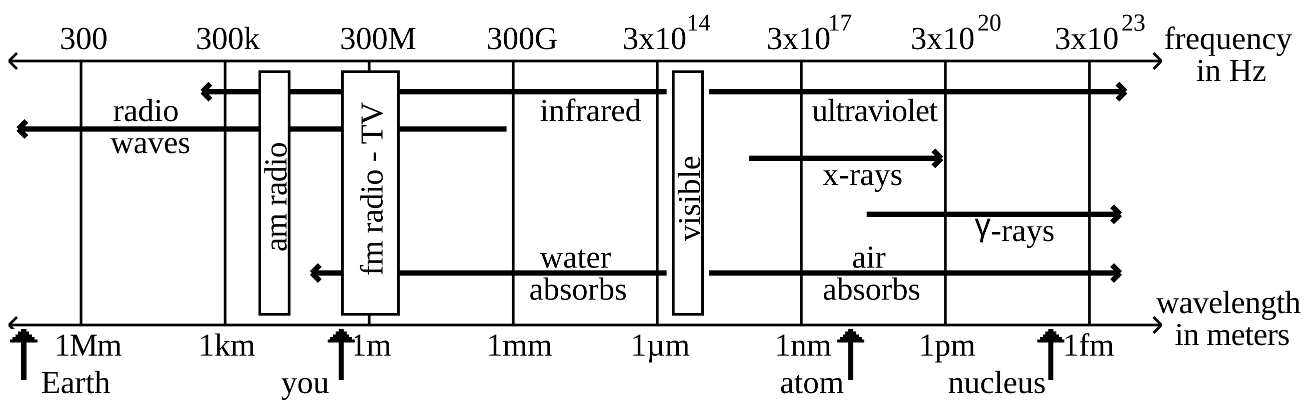
Recall that earlier we started with Faraday's Law and considered a path in the x-z plane and got the relationship, $\frac{\partial E}{\partial z} = -\frac{\partial B}{\partial t}$. Using the waves $E = E_m \sin(kz - \omega t)$ and $B = B_m \sin(kz - \omega t)$ we get, $kE_m \cos(kz - \omega t) = +\omega B_m \cos(kz - \omega t)$ $\quad \frac{E_m}{B_m} = \frac{\omega}{k} = c$.

$$\boxed{E_m = cB_m \quad \text{The Ratio of the Peak Fields}}$$

All the properties of electromagnetic waves that were stated earlier are consistent with Maxwell's Equations.

4. The Electromagnetic Spectrum

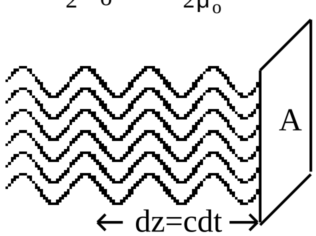
We might suspect that all waves that travel at the speed of light are in fact electromagnetic in nature. For the most part this turns out to be true. The only essential difference between different types of EM waves is the wavelength (or frequency).



Discuss: The origin of human vision and the safety of exposure to EM waves (move on to energy).

5. The Poynting Vector

Notice that $\vec{E} \times \vec{B}$ point in the direction of the velocity vector. The magnitude of $|\vec{E} \times \vec{B}| = EB = \frac{1}{c}E^2 = cB^2$. Recall that the energy density in electric and magnetic fields is given by, $u = \frac{1}{2}\varepsilon_o E^2 + \frac{1}{2\mu_o}B^2$. Therefor $\vec{E} \times \vec{B}$ must be related to the energy in the waves.



The exact relationship can be found by finding the total energy deposited by an EM wave landing on surface in a time dt. This energy must equal the energy density in the waves times the volume they occupy, $U = u\,vol \quad dU = udV = uAdz = ucAdt$. Substituting for u, $dU = \left(\frac{1}{2}\varepsilon_o E^2 + \frac{1}{2\mu_o}B^2\right)cAdt$. The power per unit area is, $\frac{dU}{Adt} = \left(\frac{1}{2}\varepsilon_o cE^2 + \frac{c}{2\mu_o}B^2\right)$. Using the result from above $|\vec{E} \times \vec{B}| = \frac{1}{c}E^2 = cB^2$ the power per unit area can be written as $\frac{dU}{Adt} = \left(\frac{1}{2}\varepsilon_o c^2|\vec{E} \times \vec{B}| + \frac{1}{2\mu_o}|\vec{E} \times \vec{B}|\right) = \left(\frac{1}{2}\varepsilon_o c^2 + \frac{1}{2\mu_o}\right)|\vec{E} \times \vec{B}|$. The two terms in parentheses are equal because $c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$ so finally, $\frac{dU}{Adt} = \frac{1}{\mu_o}|\vec{E} \times \vec{B}|$. The Poynting Vector is defined accordingly,

$$\boxed{\vec{S} \equiv \frac{1}{\mu_o}\vec{E}\times\vec{B} \quad \text{The Definition of the Poynting Vector}}$$

The magnitude of the Poynting vector is power per unit area and it points in the same direction as the velocity.

The instantaneous power per unit area is, $S = \frac{1}{\mu_o}EB = \frac{1}{\mu_o}E_m B_m \sin^2(kz - \omega t)$. Often the average power per unit area is called the "intensity." Since the average value of the square of the sine is one half,

$$\boxed{I = \frac{E_m B_m}{2\mu_o} = \frac{E_m^2}{2\mu_o c} = \frac{cB_m^2}{2\mu_o} \quad \text{Intensity of EM Waves}}$$

> <u>*Example 5:*</u> *Sunlight strikes earth with an average intensity of 1400W/m^2. Find the peak electric and magnetic fields.*
> The intensity of EM waves is, $I = \frac{E_m^2}{2\mu_o c}$ $\quad E_m = \sqrt{2\mu_o cI} = \underline{\underline{1060\,\frac{V}{m}}}$.
> The ratio of the peak fields gives, $E_m = cB_m$ $\quad B_m = \frac{E_m}{c} = \underline{\underline{3.53\mu T}}$.

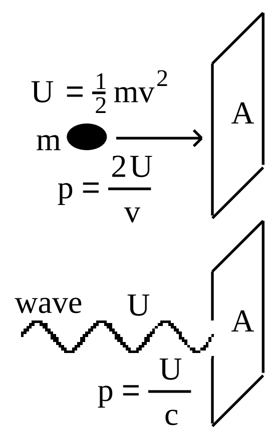
Discuss: hood of a car in the sun and issues related to solar energy.

> <u>*Example 6:*</u> *Find the total power radiated by the sun.*
> There are 1400W/m^2 landing on the earth which is $1.50\text{x}10^{11}$m away. Since the sun radiates EM waves uniformly in all directions this intensity will be the same over the surface area of a sphere that is $1.50\text{x}10^{11}$m away.
> Using the definition of power and the definition of intensity, $I \equiv \frac{dU}{Adt} = \frac{P}{A} = \frac{P}{4\pi r^2}$.
> Solving for the power, $P = I4\pi r^2 = (1400)(4\pi)(1.50\text{x}10^{11})^2 = \underline{\underline{3.96\text{x}10^{26}\,W}}$.

<u>6. Radiation Pressure and Momentum Transfer</u>

When EM waves are absorbed by a surface, not only do they deposit energy, but they transfer momentum as well. The illustration at the right shows particles with kinetic energy $U = \frac{1}{2}mv^2 = \frac{1}{2}pv$ absorbed by a surface, the momentum transferred can be written as,

$$p = \frac{2U}{v}.$$



The relationship between the transferred momentum and deposited energy for EM wave differs from this result by a factor of two. This is because the definition of kinetic energy $U = \frac{1}{2}mv^2$ is invalid for objects moving near the speed of light. For EM waves the answer is,

$$\boxed{p = \frac{U}{c} \quad \text{The Momentum Transfer for Complete Absorption}}$$

Usually it is more convenient to talk about the "Radiation Pressure" on a surface. Pressure is defined as force per unit area. Since the force on a surface is equal to the rate at which it collects momentum, $P \equiv \frac{F}{A} = \frac{dp}{Adt} = \frac{dU}{cAdt} = \frac{I}{c}$.

$$\boxed{P = \frac{I}{c} \quad \text{The Radiation Pressure for Complete Absorption}}$$

For totally reflected waves the momentum transfer and the radiation pressure are just doubled, the same as for perfectly reflected particles.

__Example 7:__ Find the force due to the radiation pressure on a perfectly reflecting automobile hood 1.00m² in area.

The initial momentum of the incident waves is $p_i = -\frac{U}{c}$.

The final momentum of the reflected waves is $p_f = \frac{U}{c}$.

The momentum transferred is, $\Delta p = p_f - p_i = 2\frac{\Delta U}{c}$ $\quad \frac{\Delta p}{\Delta t} = \frac{2}{c}\frac{\Delta U}{\Delta t}$.

This is the force on the hood of the car.

Since the Poynting Vector is the energy per second per area,

$$\frac{\Delta p}{\Delta t} = \frac{2}{c} IA = \frac{2}{3.00x10^8}(1400)(1.00) = \underline{\underline{9.33x10^{-6} N}}.$$

__Chapter 34 - Summary__

The Properties of Electromagnetic Waves

- They continue to travel after the source is turned off.
- They travel through empty space.
- They always travel at the same constant speed.
- The electric field is always perpendicular to the magnetic field.
- The velocity is perpendicular to both the electric field and the magnetic field.
- The ratio of the peak electric field to the peak magnetic field equals the speed of the waves.

The Speed of EM Waves $c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$

The Ratio of the Peak Fields $E_m = cB_m$

The Definition of the Wave Number $k \equiv \frac{2\pi}{\lambda}$

The Frequency-Wavelength Relationship $\frac{\omega}{k} = \lambda f = c$

The Definition of the Poynting Vector $\vec{S} \equiv \frac{1}{\mu_o}\vec{E} \times \vec{B}$

The Intensity of EM Waves $I = \frac{E_m B_m}{2\mu_o} = \frac{E_m^2}{2\mu_o c} = \frac{cB_m^2}{2\mu_o}$

The Momentum Transfer for Complete Absorption $p = \frac{U}{c}$

The Radiation Pressure for Complete Absorption $P = \frac{I}{c}$