Problem 7.14 Comments

1. Use the ground state hydrogen wave function as the trial function but replace the Bohr radius a with a variational parameter, b,

$$\psi = (\pi b^3)^{-\frac{1}{2}} e^{-\frac{r}{b}}.$$

2. You should find $\langle H \rangle = E_1 \left\{ -\left(\frac{a}{b}\right)^2 + 2\left(\frac{a}{b}\right)\left(1 + \frac{\mu b}{2}\right)^{-2} \right\}$ where E_1 is the ground state Bohr energy.

3. The condition that minimizes <H> can be expanded in powers of the small quantity μb giving the result,

$$a \approx b\left[1 - \tfrac{3}{4}(\mu b)^2\right]$$

to order $(\mu b)^2$.

4. You could solve the cubic equation for b. On the other hand, you could notice that a is only slightly different from b. That is to say,

$$a = b(1-\delta) \text{ or } \delta = \tfrac{3}{4}(\mu b)^2.$$

Since $a = b(1-\delta)$ implies $b = a(1+\delta)$,

$$b = a\left[1 + \tfrac{3}{4}(\mu b)^2\right] = a\left[1 + \tfrac{3}{4}\left(\mu a[1+\delta]\right)^2\right].$$

Expanding to zeroth order in δ,

$$b = a\left[1 + \tfrac{3}{4}(\mu a)^2\right].$$

5. Using this result you can find the minimum <H>.