Problem 9.3 – Explaining the hint

1. The hint suggests writing the delta function as a rectangle such as,

$$\delta(t) = \begin{cases} \frac{1}{2\varepsilon} & -\varepsilon < t < \varepsilon \\ 0 & otherwise \end{cases} .$$

We'll eventually let ε go to zero.

2. Now the coefficients can be written as,

$$\begin{array}{ccc} c_a(t) = 1 & c_b(t) = 0 & t < -\varepsilon \\ \dot{c}_a = -\frac{i\alpha}{2\varepsilon\hbar} e^{-i\omega_o t} c_b & \dot{c}_b = -\frac{i\alpha^*}{2\varepsilon\hbar} e^{i\omega_o t} c_a & -\varepsilon < t < \varepsilon \\ c_a(t) = a & c_b(t) = b & t > \varepsilon \end{array} .$$

3. Combine the coupled equations to create a second order equation for c_b. Solve this equation for c_b, then substitute to get the solution for c_a.

4. Now, set t = ε in the solutions and let ε go to zero to find a and b.