The EPR Experiment

The article "Is The Moon There When Nobody Looks ? Reality and Quantum Theory" by N. David Mermin (Physics Today, v38, pp 38-47, 1985) contains the cleanest description of an experiment that exhibits different predictions for a "complete" as opposed to the quantum theory. The exercises below are designed to illustrate the details of the arguments presented in this paper. You can get a copy at http://phys.csuchico.edu:16080/kagan/435A/problems/Mermin.pdf

The experiment consists of a source of correlated (opposite) spin 1/2 particles (e.g.. electrons) in the singlet state heading in opposite directions along the y-axis toward identical detectors. The detectors measure the spin (polarization) of the particles along one of three axes oriented at $\alpha, \beta,$ and γ at 0˚, 120˚, and 240˚ to the vertical (z-axis) respectively. Assume the axes about which the spin will be measured are chosen randomly during the flight of the particles. This choice is made independently for each detector. The detector responds by indicating whether the spin is up or down along the chosen axis. By working the exercises below the fraction of the time that the detectors give opposite readings will be found. The prediction of the complete theory and the prediction of the quantum theory will differ. Therefore, an experimental result will doom one or the other.

1. The Predictions of the Complete Theory

(a)Since each particle must know how it will respond to any of the three possible orientations of the detector, it must carry with it instructions. Such an instruction set can be indicated by three arrows, such as, $\uparrow\downarrow\downarrow$, where the order indicates the result for the first, second, and third axis respectively. Make a list of all possible instruction sets for a single particle.

(b)Make a table of allowed pairs of instruction sets keeping in mind that the detectors must give opposite readings if they are set identically.

(c)List the possible combinations of pairs of axes.

(d)Add to the table of part (b) a column and list in this column the pairs of axes that give opposite spin for each pair of instruction sets.

(e)Using the fact that all axis settings are determined randomly, find the percentage of opposite spin results for a very large number of measurements.

2. The Predictions of the Quantum Theory

(a)Find the operator that represents spin along each of the three axes by taking the dot product between a unit vector along the axis and the spin vector.

(b)Find the eigenvalues and eigenvectors for each spin operator.

(c)Express the α-axis eigenvectors as linear combinations of the eigenvectors for the two other axes.

(d)Write a two particle wave function that will always produce opposite spins if the detectors are both set to α and still will produce 50% up and 50% down on any one of the detectors set to α. Make sure it is the state with a total spin of zero (the so-called singlet state).

(e)Be sure this is a valid wave function you must always get opposite spins when the detectors are set to $\beta\beta$ or $\gamma\gamma$. Check this by using the results of part (c) express the wavefunction in such a way that you can find the probability of oppose spin for these two axis settings.

(f)Find the probability that the detectors will give opposite readings for the remaining axis pairs.

(g)Assuming the axes are set randomly, find the probability the detectors give opposite readings regardless of the axis settings.

(h)Summarize the point of this exercise.

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Hints and Answers

1a. 8 possible sets   1b. Still only 8 sets !   1c. 9 combinations

1d. 2 sets have 9 of 9, the remaining 6 sets have 5 of 9   1e. 67%

2a. For example $\hat{\beta} = \cos 30^\circ \hat{i} - \sin 30^\circ \hat{k} = \frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{k}$ giving $S_\beta = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$.

2b. For S_β, $\lambda = +1 \Rightarrow \chi_+^\beta = |\uparrow_\beta\rangle = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$ and $\lambda = -1 \Rightarrow \chi_-^\beta = |\downarrow_\beta\rangle = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{pmatrix}$

2c. For example, $|\uparrow_\alpha\rangle = \frac{1}{2}|\uparrow_\beta\rangle + \frac{\sqrt{3}}{2}|\downarrow_\beta\rangle$

2d. $|\psi\rangle = \frac{1}{\sqrt{2}}\left\{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right\} = \frac{1}{\sqrt{2}}\left\{|\uparrow_\alpha\rangle|\downarrow_\alpha\rangle - |\downarrow_\alpha\rangle|\uparrow_\alpha\rangle\right\}$

2e. For example with the switches set to ββ some work will reveal,
$|\psi\rangle = \frac{1}{\sqrt{2}}\left\{|\downarrow_\beta\rangle|\uparrow_\beta\rangle - |\uparrow_\beta\rangle|\downarrow_\beta\rangle\right\} \Rightarrow P = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$

2f. For example with the switches set at αβ,
$|\psi\rangle = \frac{1}{\sqrt{2}}\left\{\frac{\sqrt{3}}{2}|\uparrow_\alpha\rangle|\uparrow_\beta\rangle - \frac{1}{2}|\uparrow_\alpha\rangle|\downarrow_\beta\rangle - \frac{1}{2}|\downarrow_\alpha\rangle|\uparrow_\beta\rangle - \frac{\sqrt{3}}{2}|\downarrow_\alpha\rangle|\downarrow_\beta\rangle\right\}$
By examining the coefficients the probability of opposite readings is, $P = \left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2 = \frac{1}{4}$.

2g. Six pairs of setting have a 25% chance and three pairs have a 100% chance so overall probability for opposite spins is 50%.