Feynman's Proof of Maxwell's Equations

Get a copy of the paper "Feynman's Proof of Maxwell's Equations" by Freeman Dyson published in AJP **58** (3) March 1990. This proof has many interesting points, not the least of which is that Maxwell's Equations appear to be derivable from Newton's Second Law and the basic commutation relations of position and momentum from quantum mechanics. From a more modern perspective, the proof demonstrates the validity of the replacement, $\vec{p} \to \vec{p} - q\vec{A}$ in Schrodinger's Equation. Be advised that the duplicate subscript summation convention is used throughout this paper. Some properties of the Levi-Civita symbol are listed below, as are some properties of commutators.

Begin by using the Newton's Second Law (equation 1) and the basic commutation relations (equations 2 and 3) to develop constraints on the functional form of the force.

(a)Differentiate equation 3 and use equation 1 to derive equation 9.

(b)Use equation 10 and equation 3 to derive equation 11. Show how equation 11 allows F_k to be any function of coordinates or any function of the $\dot{x}$'s to the first power.

(c)Swap indices in equation 9 to get equation 12. Show how equation 12 requires that the jth component of the force can only be a function of the non-j components of the $\dot{x}$'s. From the constraints from equations 11 and 12 explain why the force must have the form suggested by Lorentz,

$$F_j = E_j + \varepsilon_{jk\ell}\dot{x}_k H_\ell,$$

where E_j and H_ℓ are only functions of position and time as shown in equations 14 and 15.

It now remains to find the constraints on the functional forms of E_j and H_ℓ.

(d)Substitute the Lorentz Force (equation 4) into equation 12 to get conditions on H (equation 13).

(e)Use equation 9 and equation 13 to get equation 16.

(f)Note that equation 16 says that the $\dot{x}$'s don't commute. In order to understand the relationship between $\dot{x}$ and space derivatives, use the fact that $\left[x_i, f(x_j,t)\right]=0$ for any function of coordinates and time, to show that $\left[\dot{x}_i, f(x_j,t)\right] = -\frac{i\hbar}{m}\frac{\partial f}{\partial x_i}$.

(g)Prove equation 17 by using a Jacobi Identity.

(h)Commute equation 16 with $\dot{x}_\ell$ and use equation 17 and to derive equation 18. Using the result of part f, show that this is equivalent to equation 5.

(i)Start with equation 16 to derive equation 6.

(j)Explain how this shows that $\vec{p} \to \vec{p} - q\vec{A}$ is valid by substituting equation 23 into equation 16.

Properties of the Levi-Civita symbol:

$\delta_{ij}\varepsilon_{ijk} = 0$ $\quad$ $\varepsilon_{ijk} = \varepsilon_{jki} = \varepsilon_{kij}$

$\varepsilon_{ijk}\varepsilon_{ij\ell} = 2\delta_{k\ell}$ $\quad$ $\varepsilon_{ijk} = -\varepsilon_{jik}$

$\varepsilon_{ijk}\varepsilon_{ijk} = 6$

$\varepsilon_{ijk}\varepsilon_{i\ell m} = \delta_{j\ell}\delta_{km} - \delta_{jm}\delta_{k\ell}$

Properties of Commutators

Commutation [A,B] = -[B,A]

The Jacobi Identity $[A,[B,C]]+[B,[C,A]]+[C,[A,B]]=0$

Multiplication Rule [AB,C]=A[B,C]-[C,A]B