435B - Phase Shift Analysis

1. Start with Eq. 11.29 and write it for large r. Now, expand the complex exponentials in terms of trig functions to get,

$$\psi(r,\theta) = A\sum_{\ell} \frac{2\ell+1}{kr} \cdot i^{\ell}\left\{ka_{\ell}\cos(kr - \ell\tfrac{\pi}{2}) + \left[1 + ika_{\ell}\right]\sin(kr - \ell\tfrac{\pi}{2})\right\}P_{\ell}(\cos\theta)$$

2. Use the trig identity, $A\cos\theta + B\sin\theta = \sqrt{A^2 + B^2}\,\sin(\theta + \gamma)$ to rewrite the wave function in terms of δ_ℓ defined as,

$$\tan\delta_{\ell} \equiv \frac{ka_{\ell}}{1 + ika_{\ell}}.$$

The result is a wave function with a phase shift,

$$\psi(r,\theta) = A\sum_{\ell} \frac{2\ell+1}{kr} \cdot i^{\ell}\sqrt{1 + 2ika_{\ell}}\,\sin(kr - \ell\tfrac{\pi}{2} + \delta_{\ell})P_{\ell}(\cos\theta)$$

3. Express a_ℓ in terms of δ_ℓ to get Eq. 11.46.