Colliding Magnetic Pendula: When Is a Collision Not Collision-like?

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One of the authors sat down on a chilly winter morning with a steaming cup of coffee and his favorite catalog.[1] There it was! Not a toasty sweater or tropical vacation, but the MagnaSwing[2] (pictured in Fig. 1). The MagnaSwing is identical to the Newtonian Demonstrator[3] (the well-known apparatus consisting of five steel balls suspended from strings), except the balls are replaced by magnets arranged so that nearest neighbors repel one another. Even though the Newtonian Demonstrator and the MagnaSwing appear to be similar devices, their motions are dramatically different. This will be explained by first reviewing the behavior of the Newtonian Demonstrator and describing the behavior of the MagnaSwing. Then we will review a two-ball collision on the Newtonian Demonstrator and describe a two-magnet collision on the MagnaSwing. Finally, we will explain why hard-sphere collisions and magnetic collisions produce different behaviors in these apparently similar devices.

Fig. 1. A photograph of the MagnaSwing.

A Review of the Newtonian Demonstrator

The history of the Newtonian Demonstrator actually goes back to the time of Newton.[4] In addition, there is a rich collection of literature in AAPT journals.[5–7] The most entrancing behavior of the device is the fact that when any number of balls are pulled upward on one side and released to collide with the remaining balls, the collision always results in the same number of balls swinging upward on the opposite side as were released. The more one thinks about this, the more amazing it is.

Consider the case of one ball released to collide with the remaining four balls. If the four balls are treated as a single mass with four times the inertia of the single ball, then conservation of momentum requires

$$mv_0 = 4mu_2 - mu_1 \Rightarrow v_0 = 4u_2 - u_1, \qquad (1)$$

while the conservation of mechanical energy requires,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}4mu_2^2 + \frac{1}{2}mu_1^2 \Rightarrow v_0^2 = 4u_2^2 + u_1^2, \quad (2)$$

where m is the mass of a single ball, v_0 is the initial velocity of the single ball, u_1 is the velocity the single ball rebounds with after the collision, and u_2 is the velocity of the four balls treated as a unit. The algebra gives the result

$$u_1 = \frac{3}{5}v_0 \text{ and } u_2 = \frac{2}{5}v_0. \tag{3}$$

This is not what happens at all, yet it is perfectly consistent with conservation of momentum and energy. The error is in considering the four balls as a unit. In fact, stating the problem more correctly: Given the initial velocities of all five balls, find the final velocities of all five balls. Now there are five unknowns. Since conservation of energy and momentum provide only two equations, they are not sufficient to uniquely determine the outcome. There is, in fact, an infinite collection of solutions consistent with these two principles. The amazing outcome of the single ball leaving and the other four staying at rest can only be explained by a detailed examination of the forces that the balls exert on each other. Several papers have discussed this issue in some detail.[5–7]

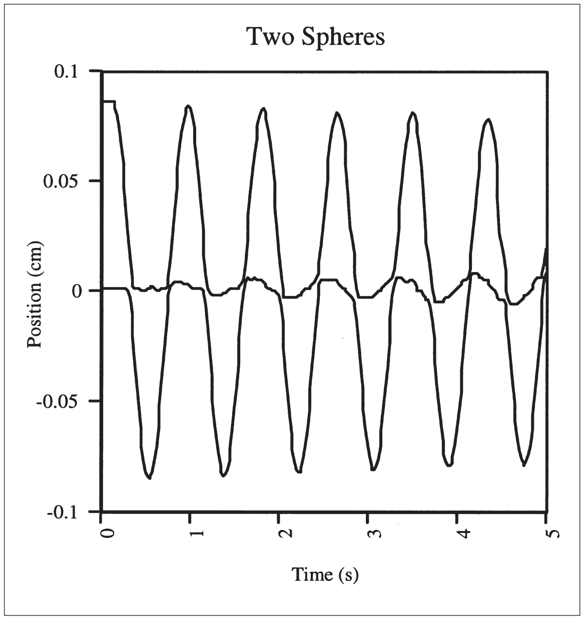

Fig. 2. Position-vs-time graph for two colliding spheres.

The Behavior of the MagnaSwing

Once the MagnaSwing arrived, we tore it out of the box, assembled it, and attempted the experiment described in the previous section. The first swing of a single magnet colliding with the remaining four magnets resulted in the last magnet getting most but not all of the energy. The magnets in between did not come to rest. Subsequent swings distributed the energy to all of the magnets, resulting in a wonderfully rich variety of motion quite unlike the Newtonian Demonstrator. This indicates that the amazing motion of the five balls in the Newtonian Demonstrator is, in fact, dependent upon the precise nature of the forces between the balls.

The magnets remain in motion for many minutes, certainly as long as the steel balls in the Newtonian Demonstrator. This shows that mechanical energy in the MagnaSwing is conserved, or more precisely, lost to heat at a rate comparable to that of the Newtonian Demonstrator. The conservative nature of the MagnaSwing's motion is not surprising in light of the conservative nature of the magnetic force. In addition, we wish to note that further experiments, not discussed here, indicate that the motion is not chaotic but instead well explained by standard normal mode methods.

The Collision Between Two Balls

To better understand the differences between the collisions in the MagnaSwing and those of the Newtonian Demonstrator, notice that in a collision between just two balls, the outcome is uniquely determined by the conservation of momentum and mechanical energy because there are only two unknown final velocities.

Using PASCO motion detectors and LoggerPro software from Vernier, we simultaneously measured the motion of each ball starting with both pendula at rest; one hanging down at equilibrium and the other pulled away from equilibrium. Figure 2 shows the motion of the two (Newtonian Demonstrator) steel balls.

For clarity we have defined the upward swing of one of the pendula to be in the negative direction, and have set the zero position where the ball is hanging vertically. The balls show the expected behavior with the oscillatory motion being exchanged between them; in fact, if one squints while looking at this graph, a nearly con-

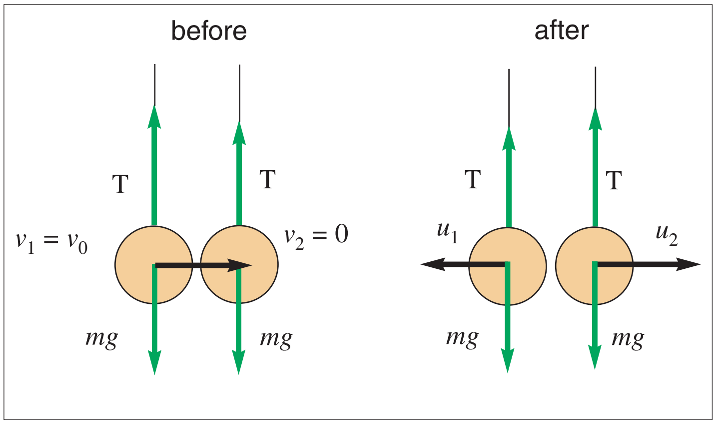

Fig. 3. The collision of the spheres.

tinuous large sinusoid appears. After a few collisions, it becomes apparent that the incoming ball does not come to rest after the collision, but continues to travel a small distance past equilibrium. This is due to slight misalignment in the strings.

In Fig. 3 it can be seen that the external forces that act during the collision are perpendicular to the motion at the time of the collision so they provide no impulse during the collision. Therefore, the unknown final velocities u_1 and u_2 can be found in terms of the initial velocities $v_1 = v_0$ and $v_2 = 0$, using the laws of conservation of momentum and energy,

$$mv_0 = mu_2 - mu_1$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mu_1^2 + \frac{1}{2}mu_2^2.$$

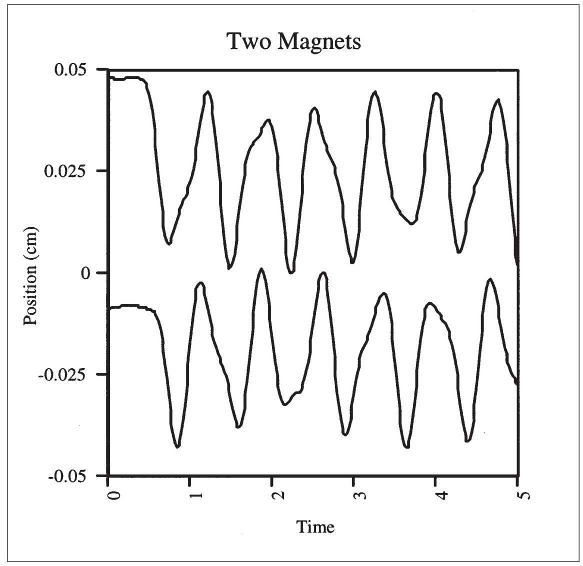

Fig. 4. Position-vs-time graph for two colliding magnets.

Solving for the final velocities gives the expected results, $u_1 = 0$ and $u_2 = v_0$. In summary, we are able to describe the motion of the two balls without understanding the details of the collision force. This, of course, we happily point out to our students, hoping they come to appreciate the surprising predictive power of conservation laws.

The Collision Between Two Magnets

Figure 4 is a graph of position versus time for several collisions of two magnets. Instead of acting like the collision between two balls, where the first ball comes essentially to rest and the second ball moves off with almost the same velocity as the first had before the collision, the magnets behaved in a more complex manner. This was particularly puzzling since the motion for any two objects was presumed to be independent of the details of the interaction force, being instead completely determined by the conservation laws.

The magnets' motion is not so readily grasped from the graph. For the time being, we simply point out that the magnets "push on" each other during the entire motion. For example, even when one magnet is initially raised to one side ($t \sim 0$), the second magnet is displaced from equilibrium.

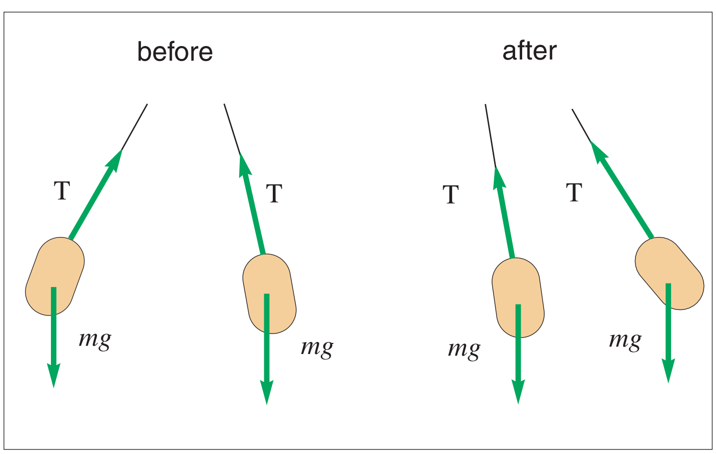

Fig. 5. The collision of the magnets.

Figure 5 illustrates the fact that the magnets are already interacting and the "collision" is occurring well before the released magnet has reached the bottom of its swing. Therefore, the external forces of gravity and tension act during the "collision" and the law of conservation of momentum cannot be used. Since the law of conservation of momentum can't provide the second equation, the details of the behavior of the forces are needed to describe the motion completely.

Note that momentum-conserving collisions can be realized when magnetic forces are involved, for instance colliding magnetic gliders on air tracks. Just be sure to keep the track horizontal!

Comments

In summary, the beautifully intricate motion of the MagnaSwing is a reminder that the interaction we typically call a collision is one in which the law of conservation of momentum can be applied. The application of this law requires no net external force on the system. Since this situation is rarely realized in practice, either the time for the interaction must be short enough that the impulse generated by the external forces can be neglected or these forces must be oriented in such a way as to exert no net impulse during the collision.

References

1. American Science Surplus has an eccentric assortment of science materials in a monthly catalog that are described in a very entertaining manner. Readers can order a catalog or browse the company's wares at http://www.sciplus.com.
2. This device is marketed by Fascination Toys and Gifts Inc., Seattle, WA 98148; 206-244-9834. American Science Surplus sold this item in their April catalog for $13.95 plus shipping.
3. This is the name used by CENCO, http://www.cenconet.com; however, in the literature, it is also known by other names such as the ball-chain, the impact ball apparatus, and Newton's Cradle.
4. Willem Jacob's Gravesande [***AU: is this author name correct?***], *Mathematical elements of natural philosophy confirm'd by experiments or, An introduction to Sir Isaac Newton's philosophy*, 2nd ed. translated by J.T. Desaguliers, (J. Senex and W. Taylor, London, 1721–1726). Available through Landmarks of Science (Readex Microprint, New York, 1969).
5. F. Herrmann and P. Schmälzle, "Simple explanation of a well-known collision experiment," *Am. J. Phys.* **49**, 761–764 (Aug. 1981).
6. F. Herrmann and M. Seitz, "How does the ball-chain work?" *Am. J. Phys*. **50**, 977–981 (1982).
7. J.D. Gavenda and J.R. Edgington, "Newton's Cradle and scientific explanation," *Phys. Teach*. **35**, 411–417 (Oct. 1997). This paper contains a very complete list of additional references on this topic.