Happy Balls, Unhappy Balls, and Newton's Cradle

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The intricacies of Newton's Cradle[1] are well covered in the literature[2-4] going as far back as the time of Newton![5] These discussions generally center on the highly elastic collisions of metal spheres. Thanks to the invention of happy and unhappy balls,[6] you can build and study the interaction of less elastic systems (see Fig. 1).

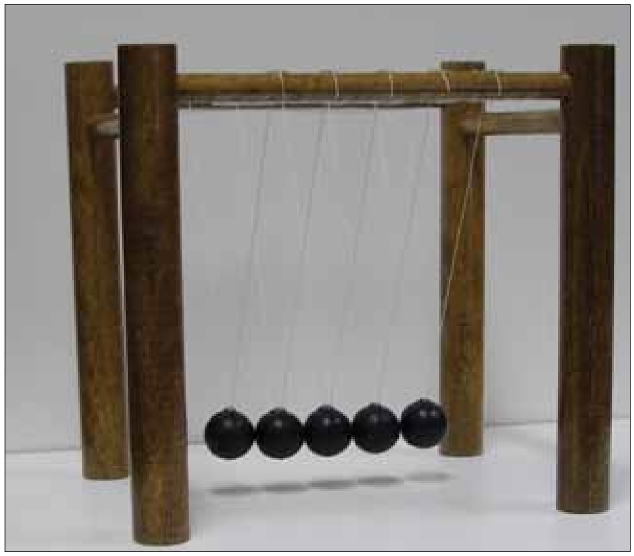

Fig. 1. Happy (or unhappy) balls in Newton's Cradle.

The elasticity of a collision is described by the coefficient of restitution (COR), which is the ratio of the outgoing relative velocity to the incoming relative velocity. The literature that accompanies the happy and unhappy balls claims the happy ball has a COR of 0.53 and the unhappy ball 0.03. The steel balls appear to have a COR of greater than 0.95.

The physics of a collision between two equal mass balls is straightforward. If the incoming ball has a speed v_0 and the second ball starts at rest, the law of conservation of momentum requires

$$v_0 = v_1 + v_2, \tag{1}$$

where v_1 is the speed of the incoming ball after collision and v_2 is the speed of the second ball after collision. The definition of COR requires

$$\varepsilon = \frac{v_2 - v_1}{v_0} \Rightarrow \varepsilon v_0 = v_2 - v_1. \tag{2}$$

Solving Eqs. (1) and (2) for the speeds of the balls after collision yields

$$v_2 = \tfrac{1}{2}(1+\varepsilon)v_0 \text{ and } v_1 = \tfrac{1}{2}(1-\varepsilon)v_0. \tag{3}$$

For the case of the perfectly elastic collisions ($\varepsilon = 1$), we get the expected result that the incoming ball comes to rest while the second ball heads off at the same speed as the incoming ball had before the collision.

For inelastic collisions ($\varepsilon < 1$) we get the incoming ball following the second ball after the collision. As the COR decreases, the incoming ball follows at increasing speed until both balls head off together at half the initial speed when the COR reaches zero. Notice that when the incoming ball collides with more than one ball, the law of conservation of momentum and the definition of COR are no longer sufficient to determine the resulting motion of all the balls.

If you pull one ball to the side and release it in the five-ball system, what sort of motion do you expect? I hate to spoil the surprise by telling you too much, but in the end the happy balls, the unhappy balls, and the steel spheres all exhibit the same motion. All five spheres wind up swinging in unison. For the highly elastic metals spheres it takes minutes to reach this state. The happy balls take much less time. The unhappy balls? You guessed it! It happens after the first collision.

References

1. This device has many names: the Newtonian Demonstrator (Sargent-Welch, WLS1800-07), the ball-chain apparatus, the Swinging Wonder (Snowcraft, Hastings, MI), impact balls, etc.
2. F. Herrmann and P. Schmälzle, "Simple explanation of a well-known collision experiment," *Am. J. Phys.* **49,** 761-764 (Aug. 1981).
3. F. Herrmann and M. Seitz, "How does the ball-chain work?" *Am. J. Phys.* **50,** 977-981 (Nov. 1982).
4. J. D. Gavenda and J. R. Edgington, "Newton's Cradle and scientific explanation," *Phys. Teach.* **35**, 411-417 (Oct. 1997). This paper contains a very complete list of additional references on this topic.
5. Willem Jacob 'sGravesande, *Mathematical elements of natural philosophy confirm'd by experiments or, An introduction to Sir Isaac Newton's philosophy,* 2nd ed., translated by J.T. Desaguliers (J. Senex and W. Taylo, London, 1721-1726). Available through Landmarks of Science (Readex Microprint, New York, 1969).
6. Available from Sargent-Welch, product number WL0709.

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