The Problem with Some Problems

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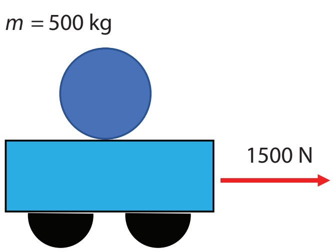
We would like to share a problem with you that has some subtle features. We will examine how we believe students may approach this problem — students that we shall characterize as "marginal," "good," and "exceptional" for convenience of discussion. We will see that one can use the incorrect response of the good student as a springboard to go into more advanced course content.

As instructors, it can be challenging to create problems, particularly multiple-choice problems, that validly examine whether our students understood what we wanted them to learn.[1] Sometimes we discover a problem that has the following properties: at first blush it appears straightforward, a deeper look makes the problem seem rather complex, and a truly insightful examination reveals that it is indeed straightforward.

These sorts of problems are dangerous if they are used for summative assessment, because both marginal students and exceptional students get the correct answer. However, reasonably good students get the wrong answer. You certainly wouldn't want poorer students appearing to be better than more skilled students. That said, problems with these properties can be quite useful for motivating future course content because of the variety of questions and discussions they can elicit.



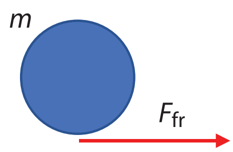
The problem

A 500-kg cylindrical bale of hay sits on a cart. When the cart is pulled to the right by a net force of 1500 N, friction exerts a force of 300 N on the bottom of the bale. The bale rolls without slipping. Find the acceleration of the bale.

A) 0 m/s^2
B) 0.2 m/s^2
C) 0.6 m/s^2
D) 1 m/s^2

A deep conceptual understanding of rolling without slipping is particularly difficult for physics students.[2] A marginal student might use some formulaic thinking and just write down the equation $F = ma$. There is only one mass given in the problem, so the only question is which force to use. There are four choices: 300 N, 1500 N, their difference, and their sum. The only one that gives one of the answers is 300 N, which results in an acceleration of 0.6 m/s^2, leading to answer C—the correct answer. The marginal student doesn't react to the complex collection of excess information presented in the statement of the question. He just grabs a formula.

The exceptional student understands that the second law requires the total force on an object is equal to the product of the mass and the acceleration of the object. She draws the free-body diagram shown below.



The second law requires $F_{fr} = ma_b$. So,

$$a_b = \frac{F_{fr}}{m}. \tag{1}$$

This student notices the excess information, but realizes these data are irrelevant. The exceptional student understands the second law thoroughly and uses the bale's mass and the 300-N force on the bale to get the same acceleration as the marginal student.

The good student gets caught up in all the excess numerical values leading to many questions. If the force is on the bottom of the bale, won't the bale rotate? If the bale rolls without slipping, do you need to worry about the connection between the motion of the center of mass and rotational motion? Since there are two objects—the bale and the cart—does the cart's motion matter? If it does, don't we need to know its mass?

At this point, many of us would simply explain the correct answer and move on to other things. However, the questions of the good student are very worthy of consideration, if only to realize they aren't relevant to the problem as asked and, more importantly, to build the deeper understanding of the second law that only the exceptional student seems to possess.

In addition, looking into these questions can quite naturally propel the class into topics that lie ahead in the syllabus anyway. So, let's examine these questions in some detail.

A deeper look

Let's look at these questions by finding the angular acceleration caused by the 300-N force on the bale using the second law for rotation. A thorough exposition of the role of friction in rolling without slipping may be found in Ref. 3.

$$rF_{fr} = I\alpha_b, \tag{2}$$

where r is the radius of the bale, I is the rotational inertia of the bale, and α_b is the angular acceleration in the bale. Using the relationship between the angular acceleration and the acceleration of the center of mass for the rolling without slipping condition, $a_b = r\alpha_b$, results in an acceleration of

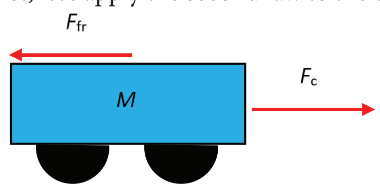
$$a_b = \frac{2F_{fr}}{m}, \qquad (3)$$

where we assume $I = 1/2\ mr^2$. The result is $a_b = 1.2\ m/s^2$, in conflict with our previous calculated acceleration of $0.6\ m/s^2$.

Since the acceleration of the bale is too small for it to be rolling without slipping, it must be slipping. Slipping with respect to what? If it is slipping, why does the problem state the bale rolls without slipping? This conundrum can be resolved if we understood the second law more deeply. In particular, the way the second law works in accelerated reference frames.

The statement of the problem must be trying to imply that the bale is rolling without slipping with respect to the cart. So, we might want to examine this system in the accelerated frame of the cart. It turns out this puts some constraints on the mass of the cart. Let's watch this play out.

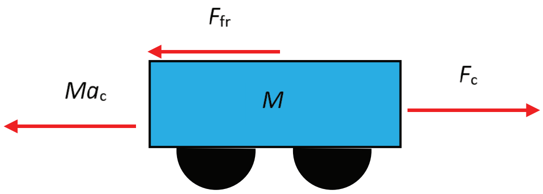
First, let's apply the second law to the cart in the lab frame.



The second law requires $F_c - F_{fr} = Ma_c$. So,

$$a_c = \frac{F_c - F_{fr}}{M}. \qquad (4)$$

The cart frame is non-inertial, so one needs to include the so-called fictitious force opposite to the acceleration of the cart in the lab frame.

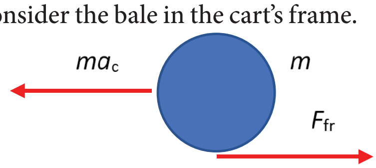


The second law requires $F_c - F_{fr} - Ma_c = Ma_c'$, where a_c' is the acceleration of the cart in the cart frame. Substituting for a_c from Eq. (4) gives

$$a_c' = 0, \qquad (5)$$

as it must, because the cart is at rest in its own frame.

Now, consider the bale in the cart's frame.



The second law requires $F_{fr} - ma_c = ma_b'$, where a_b' is the acceleration of the bale in the cart frame. So,

$$a_b' = \frac{F_{fr}}{m} - a_c. \qquad (6)$$

We need to be a bit careful here because the acceleration of the bale in this frame is negative (the bale is rolling toward the back of the cart). The second law for rotation requires

$$rF_{fr} = -\tfrac{1}{2}mr^2\alpha_b'. \qquad (7)$$

If the bale rolls without slipping in the cart frame,

$$a_b' = r\alpha_b'. \qquad (8)$$

Combining Eqs. (7) and (8),

$$F_{fr} = -\tfrac{1}{2}ma_b'. \qquad (9)$$

Switching back into the lab frame,

$$F_{fr} = -\tfrac{1}{2}m(a_b - a_c). \qquad (10)$$

Combining this with Eqs. (1) and (4),

$$F_{fr} = -\frac{1}{2}m\left(\frac{F_{fr}}{m} - \frac{F_c - F_{fr}}{M}\right), \qquad (11)$$

and solving for the mass of the cart,

$$M = \frac{1}{3}m\left(\frac{F_c}{F_{fr}} - 1\right), \qquad (12)$$

This result shows that the force on the cart must always be larger than the frictional force on the bale. This makes sense because the bale would not be able to roll without slipping without exerting more backward force on the cart than the applied forward force.

In addition, if the force on the cart gets very large, so must the mass of the cart, keeping the cart's acceleration small enough to keep the bale from slipping. Using the values given in the problem, the mass of the cart is

$$M = 670\ \text{kg}. \qquad (13)$$

Deeper learning

This problem is a case where neither the brilliance of the exceptional student nor the plug-and-chug thinking of the marginal student is as satisfying as pursuing the questions of the good student. These questions naturally advance the course toward topics that deepen the understanding of the second law specifically (reference frame dependence and its application to rotational motion) and motion in general (rolling and slipping).

Science doesn't advance by scientists remembering the "right answers" or plugging numbers into formulas. Instead science moves forward by investigating good questions. Choosing to address the questions of the good student as opposed to accepting the answers of the exceptional or marginal students reminds us of the Tao, which encourages one to take the Middle Way—the path between two ideas that seem incompatible.

References

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2. L. G. Rimoldini and C. Singh, "Student understanding of rotational and rolling motion concepts," *Phys. Rev. ST Phys. Educ. Res.* **1**, 010102 (2005).
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