

Rayleigh scattering at 70°
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Fraunhofer C to F lines.
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Among the subjects touched upon in the last chapter the first to be discussed in greater detail will be related to the sections on polarization by reflection and transmission. There we considered the effects at the particular angle of incidence called the polarizing angle. Going beyond this special case, we shall now investigate the characteristics of the reflected and transmitted light as they depend on wavelength, polarization, and angle of incidence. It will be assumed that the surfaces are optically smooth, which means that any irregularities are small compared with the wavelength. The properties of the reflecting substance play an essential role, and among these absorption is an important one. Metals are in general the best reflectors, a fact which will be found to be related to their ability to conduct electricity, and consequent high absorption. We begin, however, with the simpler case of nonconducting dielectric materials like glass.

25.1 REFLECTION FROM DIELECTRICS

The essential features of reflection from a single glass surface are briefly described as follows. At normal incidence about 4 percent of the intensity of a beam of unpolarized visible light is reflected, and the other 96 percent is transmitted. At other angles of

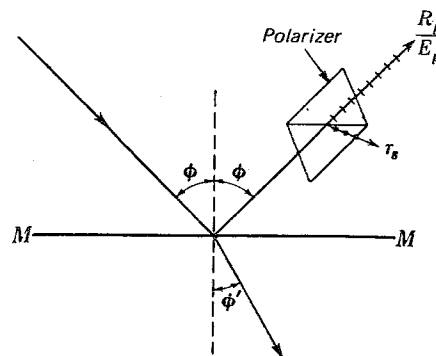


FIGURE 25A
Analysis of the reflected light into its two plane-polarized components.

incidence the reflecting power increases with angle, at first slowly and then more rapidly until at 90° , that is, grazing incidence, all the light is reflected.

It was shown at the beginning of the last chapter that there is one angle of incidence for which the reflected light is completely plane-polarized with its electric vector perpendicular to the plane of incidence. At angles different from this the reflected light is only partially polarized. The relations in this case are most easily described in terms of the reflection of the two plane-polarized components of the incident unpolarized light, the vibrations of which are, respectively, parallel and perpendicular to the plane of incidence. In the laboratory this is usually done by examining the reflected light that passes through a nicol or other polarizer (see Fig. 25A). If the polarizer is oriented with its principal section parallel to the plane of incidence, the p vibrations, i.e., the vibrations parallel to the plane of incidence, can be measured. Rotation of the polarizer through 90° then allows the s vibrations perpendicular to the plane of incidence (s stands for the German *senkrecht*, meaning *perpendicular*) to be measured. The results of such an experiment when plotted against the angle of incidence ϕ are represented by the two solid curves shown in Fig. 25B(a). The ordinates are R_p^2/E_p^2 , the fraction of the incident p light reflected, and R_s^2/E_s^2 , the corresponding fraction for the s light. These fractions are called the *reflectances* for p and s light. Part (b) of the figure refers to the amplitudes and will be discussed below.

The curves of Fig. 25B are represented very accurately by theoretical equations which were first derived by Fresnel from the elastic-solid theory and are known as *Fresnel's laws of reflection*. For the present we shall merely state them and show their application to the main features from dielectrics. The laws may be written

$$\frac{R_s}{E_s} = -\frac{\sin(\phi - \phi')}{\sin(\phi + \phi')} \quad \frac{R_p}{E_p} = \frac{\tan(\phi - \phi')}{\tan(\phi + \phi')} \quad (25a)$$

$$\frac{E'_s}{E_s} = \frac{2 \sin \phi' \cos \phi}{\sin(\phi + \phi')} \quad \frac{E'_p}{E_p} = \frac{2 \sin \phi' \cos \phi}{\sin(\phi + \phi') \cos(\phi - \phi')} \quad (25b)$$

Here the symbols E , R , and E' mean the amplitudes of the electric vectors in the incident, reflected, and refracted light, respectively, the subscripts denoting the two

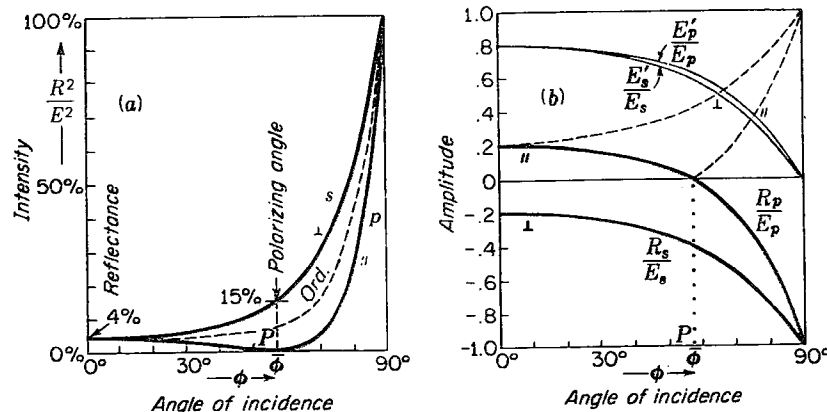


FIGURE 25B

Reflectances and the corresponding amplitudes for a dielectric having $n = 1.50$.

planes of vibration. The angles ϕ and ϕ' , following our usual notation, are the angles of incidence and refraction.

The fractional amplitudes given by Eqs (25a) and (25b) are plotted against the angle of incidence in Fig. 25B(b), the values of ϕ and ϕ' used in the equations corresponding to the index of refraction 1.50. Solid curves represent the amplitudes, both positive and negative, as they are given by the equations, while the absolute magnitudes of the reflected components are shown by broken curves. The negative signs indicate phase changes of π , which will be discussed below. These, however, are immaterial for the intensities since the latter are dependent on the squares of the amplitudes.

The reflectances are given by

$$\frac{R_s^2}{E_s^2} \quad \text{and} \quad \frac{R_p^2}{E_p^2} \quad (25c)$$

and these are the curves in part (a) of the figure. At normal incidence, where $\phi = 0$, the parallel and perpendicular components must be equally reflected because here the plane of incidence is undefined and the two components are not distinguishable. With increasing ϕ R_p^2/E_p^2 drops and R_s^2/E_s^2 rises until at the polarizing angle their values are zero and 15 percent, respectively. At grazing incidence both components are totally reflected. Even an unsilvered glass surface becomes a nearly perfect mirror when the light source is viewed very close to the reflecting plane. It is easily verified that the glaze on a page of this book becomes highly reflecting at a grazing angle.

The value of the reflectance at normal incidence does not follow immediately from Eqs. (25a) by setting $\phi = 0$, since this substitution gives an indeterminate result. It can be evaluated, however, as follows. Since both ϕ and ϕ' become small as we approach perpendicular incidence, we may set the tangents equal to the sines, obtaining

$$\frac{R_p}{E_p} = - \frac{R_s}{E_s} = \frac{\sin(\phi - \phi')}{\sin(\phi + \phi')} = \frac{\sin \phi \cos \phi' - \cos \phi \sin \phi'}{\sin \phi \cos \phi' + \cos \phi \sin \phi'}$$

Dividing numerator and denominator of the last expression by $\sin \phi'$ and replacing $\sin \phi / (\sin \phi')$ by n , we find that it reduces to

$$\frac{R}{E} = \frac{n \cos \phi' - \cos \phi}{n \cos \phi' + \cos \phi} \approx \frac{n - 1}{n + 1} \quad (25d)$$

The approximate equality becomes exact in the limit when the angles become zero. Hence the reflectance at normal incidence is

$$\frac{R^2}{E^2} = \left(\frac{n - 1}{n + 1} \right)^2 \quad (25e)$$

This very useful equation gives the reflectance at $\phi = 0$ for any single clean surface of a dielectric. Thus a glass having $n = 1.50$ has $R^2/E^2 = 0.04$, or exactly 4 percent as indicated in Fig. 25B(a).

25.2 INTENSITIES OF THE TRANSMITTED LIGHT

One might expect that the transmitted intensities would be complementary to the reflected ones, so that the two would add to give the incident intensity, but this is not so. The intensity is defined as the energy crossing unit area per second, and the cross-sectional area of the refracted beam is different from that of the incident and reflected beams except at normal incidence. It is the total energy in these beams that is complementary. There are, however, simple relations between the incident, reflected, and transmitted *amplitudes*, which follow, as we shall show later, from the boundary conditions of electromagnetic theory. These are

$$\frac{E'_s}{E_s} - \frac{R_s}{E_s} = 1 \quad \text{and} \quad n \frac{E'_p}{E_p} - \frac{R_p}{E_p} = 1 \quad (25f)$$

In Fig. 25B(b) it will be seen that the curves for E'_s and R_s run parallel to each other. Those for E'_p and R_p are not parallel but become so if the ordinates of the former curve are multiplied by n . Since Eqs. (25f) are simpler than the Fresnel equations (25b), it is sufficient to remember only the former in addition to Eqs. (25a) in order to solve problems involving transmitted amplitudes and intensities.

The fraction of the incident intensity that is transmitted, or the transmittance, when light enters a dielectric of refractive index n is not given directly by the square of the relative amplitude. The intensity in the medium according to Eq. (23g) also contains a factor n , so that the transmittance becomes $n(E'/E)^2$. Now, as stated above, the sum of this and the reflectance $(R/E)^2$ does not equal unity, as can easily be verified from Eqs. (25a) and (25b). The total energy flux in the refracted beam is its intensity times its area, and the latter differs from that of the incident or reflected beams in the ratio $(\cos \phi')/(\cos \phi)$. Conservation of energy is then expressed by the relation

$$\left(\frac{R}{E} \right)^2 + n \left(\frac{E'}{E} \right)^2 \frac{\cos \phi'}{\cos \phi} = 1$$

which applies to either s or p light.

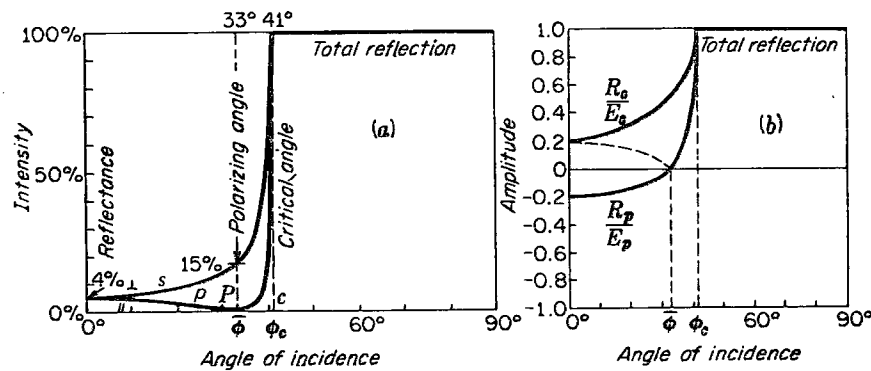


FIGURE 25C
Intensity and amplitude curves for internal reflection at a dielectric boundary,
 $n = 1.54$.

25.3 INTERNAL REFLECTION

In the above discussion it was assumed that the light strikes the boundary from the side of the optically less dense medium (usually air) so that we were dealing with the so-called rare-to-dense, or *external, reflection*. Fresnel's laws apply equally well to the case of dense-to-rare, or *internal, reflection*. If the same value of n is to be retained for the dense medium, the latter case merely involves the exchange of ϕ and ϕ' in the equations. The resulting curves are plotted in Fig. 25C, the reflectances in (a) and the amplitudes in (b). Up to the critical angle $\phi_c = 41^\circ$ these resemble the curves for external reflection, starting with $R^2/E^2 = 4$ percent at normal incidence and diverging until the polarizing angle $\bar{\phi}$ is reached. This angle, 33° , corresponds to the angle of refraction at the polarizing angle in the external case, since the angle in the rarer medium (57°) must be such as to render the refracted and reflected rays perpendicular to each other.

At the critical angle the refracted ray leaves at a grazing angle, and the internal reflectance becomes 100 percent just as for external reflection at grazing incidence. When ϕ exceeds the critical angle, Fresnel's equations contain imaginary quantities but as we shall see may still be used. It will be found that the reflection remains total but that there is a continually changing phase shift.

25.4 PHASE CHANGES ON REFLECTION

Returning for the moment to external reflection, where $\phi > \phi'$ through the whole range, we find from Eqs. (25a) that the sign of R_s/E_s is always negative. This means that there is an abrupt change of phase of 180° in the process of reflection. We express it by writing $\delta_s = \pi$. For the p light the sign is positive for small ϕ , indicating no phase change, but when the condition $\phi + \phi' = 90^\circ$ is reached, the tangent in the

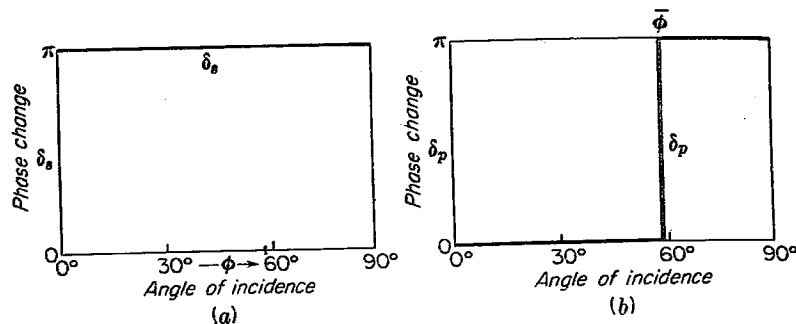


FIGURE 25D

Phase change of the electric vector of plane-polarized light externally reflected from a dielectric.

denominator goes to infinity and changes sign. Thus δ_p changes abruptly from zero to π at the polarizing angle. No real discontinuity is involved, however, because at this angle the amplitude of the p light goes through zero [Fig. 25B(b)]. Plots of δ_p and δ_s for the entire range of ϕ are given in Fig. 25D.

The directions in space of the electric vector before and after reflection are shown in Fig. 25E. It is seen that in case (a), where δ_p is taken as zero, the incident and reflected vectors are in nearly opposite directions. This apparent contradiction comes from our convention of regarding a displacement as positive or negative according as it is seen looking *against the light* in all cases. If the observer turns from viewing the incident beam to viewing the reflected beam, the rotation occurring in the plane of incidence, he finds that the two arrows maintain the same orientation relative to him. It is unfortunate that this convention gives a phase change for the s light but none for the p light at normal incidence, because at $\phi = 0$ the distinction

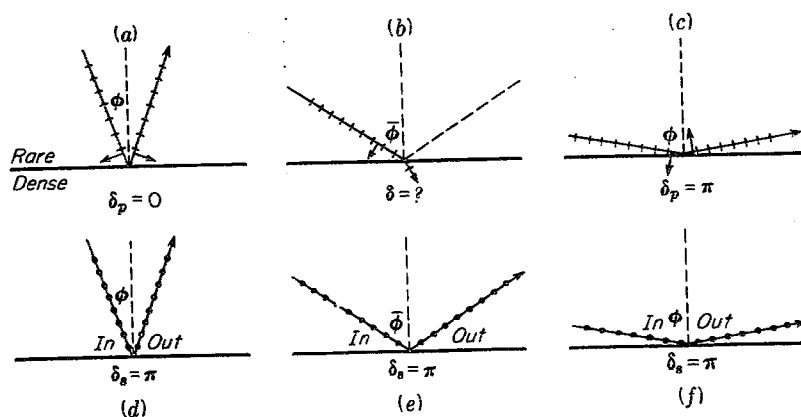


FIGURE 25E

Positions in space of the electric vector just before and just after external reflection from a dielectric.

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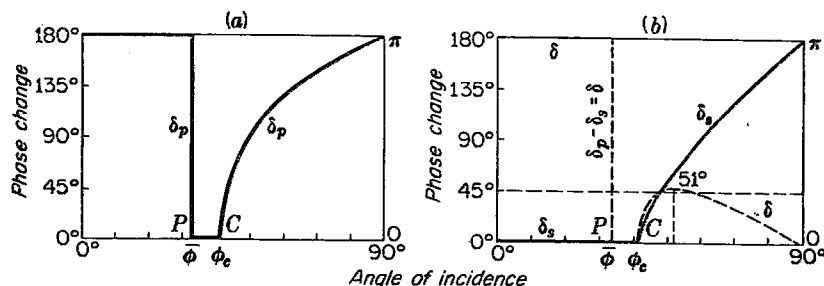
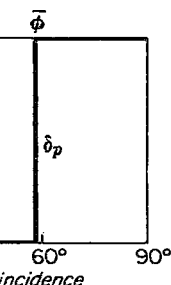


FIGURE 25F

Phase changes of the electric vector for internal reflection from a dielectric, $n = 1.51$.

between s and p vanishes. Using the opposite convention for p would, however, lead to an equally bad inconsistency in case (c) of the figure.

The phase changes that occur in internal reflection are, up to the critical angle, exactly the reverse of those at the corresponding angles for external reflection. This is a necessary consequence of Stokes' relations [Eq. (14d)], according to which there must be a relative difference of π between the two cases. Beyond ϕ_c , in the region of total reflection, Eqs. (25a) lead* to the following expressions for the tangent of half the phase change:

$$\tan \frac{\delta_s}{2} = \frac{\sqrt{n^2 \sin^2 \phi - 1}}{n \cos \phi} \quad \tan \frac{\delta_p}{2} = n \frac{\sqrt{n^2 \sin^2 \phi - 1}}{\cos \phi} \quad (25g)$$

In Fig. 25F are shown separate curves for δ_p and δ_s , and for their difference $\delta = \delta_p - \delta_s$. The δ_p curve rises more steeply than δ_s and at $\phi = 45^\circ$, according to Eqs. (25g), is exactly twice as large. Since the curves come together again at $\phi = 90^\circ$, their difference δ passes through a maximum and decreases to zero. The principle of the Fresnel rhomb (Sec. 25.6) is based on this fact.

25.5 REFLECTION OF PLANE-POLARIZED LIGHT FROM DIELECTRICS

We are now prepared to predict the nature of the reflected light when plane-polarized light is incident on the surface at any angle. Let the light fall on a plate of glass as in Fig. 25G with the plane of vibration making an angle $\psi = 45^\circ$ with the *perpendicular* to the plane of incidence.† This angle we shall call the *azimuth* angle, whether it refers to the light vibrations in the incident, reflected, or refracted beam. The incident light, of amplitude E , can here be resolved into two equal components E_p and E_s and each of these treated separately.

* See, for example, M. Born, "Optik," p. 43, J. Springer, Berlin, 1933.

† It is customary to measure ψ this way because the plane of polarization was originally defined to be at right angles to what we now call the plane of vibration.

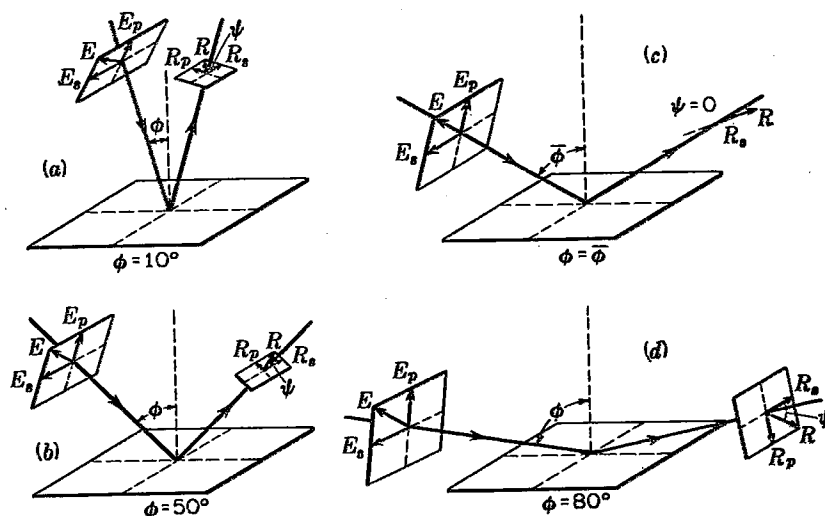


FIGURE 25G

Azimuths and amplitudes of plane-polarized light externally reflected from a glass surface at different angles of incidence.

Consider first the case where the angle of incidence ϕ is small, as in diagram (a) of the figure. Referring to Fig. 25B(b), the amplitudes of the two reflected components are found to be small and very nearly equal in magnitude. They are also out of phase by 180° . If the angle ϕ is about 10° , the component R_s is just a little larger than R_p . Taking the vector sum of the reflected components, one finds R in the direction shown. In case (b) the azimuth of the incident light is again 45° , but the angle of incidence is about 50° . R_p is now quite small and in phase with E_p , whereas R_s is larger than before and still 180° out of phase with E_s . The reflected ray remains plane-polarized, but the plane of vibration has turned farther away from the plane of incidence. When $\phi = \bar{\phi}$, as in (c), $R_p = 0$, while R_s is still larger and maintains the same phase. The resultant amplitude has continued to grow and now stands at right angles to the plane of incidence. In diagram (d), where the angle ϕ approaches 90° (grazing incidence), the reflected components have increased markedly, approaching in magnitude those of the corresponding components in the incident light. Both these components have now undergone a phase change of 180° , so that the reflected light approaches 100 percent in intensity and the plane of vibration approaches the plane of the incident light.

An equation giving the variation of the plane of vibration of the reflected light with the angle of incidence is readily obtained by dividing the two equations (25a).

$$\frac{R_p}{R_s} = - \frac{E_p \cos(\phi + \phi')}{E_s \cos(\phi - \phi')} \quad (25h)$$

This is the tangent of the angle ψ , that is,

$$\frac{R_p}{R_s} = \tan \psi \quad (25i)$$

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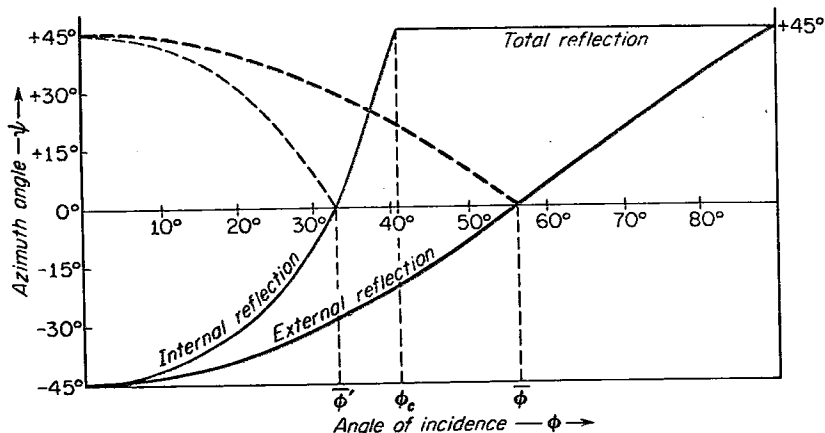


FIGURE 25H
The azimuth angle for plane-polarized light reflected from a dielectric.

since the azimuth ψ is the angle between R and R_s . This angle is plotted in Fig. 25H for the case where the incident light has an azimuth of 45° , making $E_p = E_s$. The heavily drawn curves are for external reflection, while the lighter curves, to be discussed in the following section, are for internal reflection.

25.6 ELLIPTICALLY POLARIZED LIGHT BY INTERNAL REFLECTION

Reference to Fig. 25F(b), which gives the phase change for light internally reflected from a glass surface, will show that at an angle of incidence near 50° there is a phase difference of slightly more than 45° between the two components. More exactly, the phase difference for $n = 1.51$ reaches a maximum of $45^\circ 56'$ at $\phi = 51^\circ 20'$ and is just 45° at the two angles $\phi = 48^\circ 37'$ and $54^\circ 37'$. This behavior of the phase difference was first tested and confirmed by Fresnel, who constructed a glass rhomb of the form illustrated in Fig. 25I. Plane-polarized light falls normally on the shorter surface of the rhomb with its plane of vibration oriented at 45° to the plane of the paper. It then strikes the first diagonal surface at an (internal) angle of incidence of $54^\circ 37'$. Here it is totally reflected with a phase difference of 45° between the two components. Now we have seen in Sec. 12.9 that the result of combining two linear vibrations at right angles is in general an elliptical one, the shape of the ellipse depending on the two amplitudes and on their phase difference δ . Only when δ is some integral multiple of π is the resultant linear, and the light plane-polarized. This situation exists in all cases of external reflection, and in internal reflection up to the critical angle. But in total reflection one obtains elliptically polarized light as the result of a single internal reflection at $\phi > \phi_c$. The systematic study of elliptical and circular polarization will be taken up in Sec. 27.5.

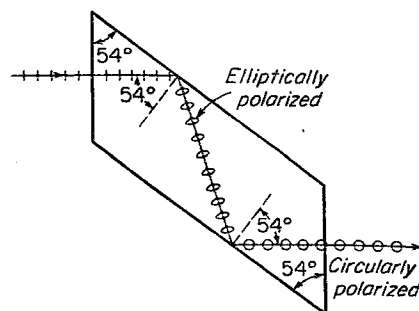


FIGURE 25I
Fresnel rhomb. The angle shown is for
glass having $n = 1.51$.

Circularly polarized light will occur only if the two amplitudes are equal and the phase difference is 90° . In the Fresnel rhomb an additional phase difference of 45° is produced by a second internal reflection, so that on emergence the p component is 90° ahead in phase. The device is therefore useful in producing and analyzing circularly polarized light, and, as we shall see later, there are other somewhat more common methods of doing this.

The polarization of the reflected beam when plane-polarized light undergoes a single internal reflection at various angles of incidence is shown in Fig. 25J. The amplitude of the electric vector in the incident and reflected light, and their components, are designated as in Fig. 25G for external reflection. Here, however, they are shown as they would appear to an observer looking against the direction of each beam, with the plane of incidence cutting the plane of the page in a horizontal line. If these diagrams are studied in connection with Figs. 25C, F, and H, their main features should

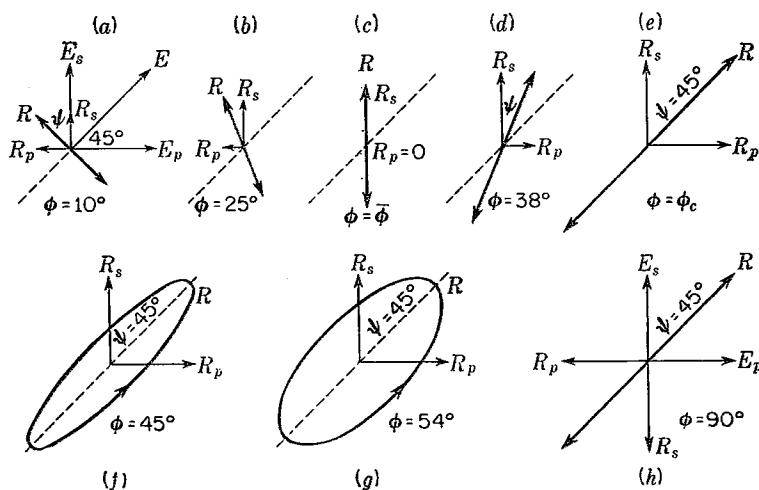


FIGURE 25J
Modes of light vibrations internally reflected in glass at various angles of
incidence.

be clear. From $\phi = 0^\circ$ to $\phi = \phi_c$ the reflected light remains plane-polarized but changes its azimuth steadily and increases in intensity. Beyond ϕ_c the vibration opens out into an ellipse having its maximum width at $\phi = 51^\circ$, then narrows again to a linear vibration at 90° .

25.7 PENETRATION INTO THE RARE MEDIUM

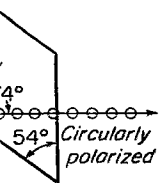
One might conclude from the fact that internal reflection beyond the critical angle is total that the amplitude of the light drops discontinuously to zero just at the reflecting surface. According to the boundary conditions of electromagnetic theory this is not possible, however, and furthermore there is experimental evidence that a disturbance capable of producing light exists for a short distance beyond the surface. Suppose that a given surface is totally reflecting an intense beam of light and one brings the edge of a razor blade very close to the surface, or scatters fine particles on it. Observation of the edge of the blade or of the particles with a microscope will show them to be secondary sources of light. In the absence of such foreign matter the electromagnetic theory predicts a disturbance which dies off exponentially beyond the surface* but which involves no net transfer of energy through it. The energy merely oscillates in and out through the surface. The disturbance is periodic in a direction parallel to the surface but not at right angles to it, and hence cannot properly be called a light wave. When the electromagnetic field is distorted by the presence of denser matter sufficiently close to the surface, however, energy is drained off in the form of light.

An instructive experiment to illustrate this penetration was performed by Hall,† who used it for quantitative measurements of the distance of penetration. The apparatus, as shown in Fig. 25K, consists of two total-reflection prisms, one of which has a slightly convex face. If the two prisms are barely in contact at the point C and the angle of incidence exceeds the critical angle, total reflection would send all the light in the direction (b). Actually it is found that in the reflected light there is a dark patch about C and a corresponding bright one in the transmitted light. Photographs of these are shown in the figure. As the angle of incidence is increased further beyond ϕ_c , the size of the patch shrinks, showing that the distance of penetration decreases. At an angle of incidence just below the critical angle (the rays indicated by broken lines), the complete set of Newton's rings in reflection and transmission appears, as illustrated by the ring patterns at the left and right of the figure. Hall used measurements of the diameters of these rings to measure the thickness of the air layers corresponding to different observed diameters of the patch mentioned above. He thus had an accurate measure of the distance of penetration. Both theory and experiment give a decrease of the energy to about $\frac{1}{100}$ in a distance of one wavelength when $\phi = 45^\circ$ and $n = 1.51$. When $\phi = 60^\circ$, it decreases to $1/40,000$ in the same distance.

* Quantitative relations are given, for example, in R. W. Ditchburn, "Light," p. 434, Interscience Publishers, Inc., New York, 1953; reprinted (paperback), 1963.

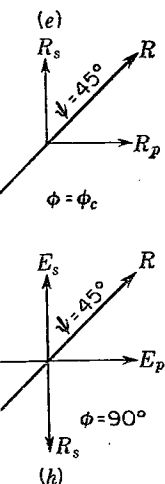
† E. E. Hall, *Phys. Rev.*, 15:73 (1902). See also K. H. Drexhage, *Monomolecular Layers and Light*, *Sci. Am.*, 222:108 (March 1970).

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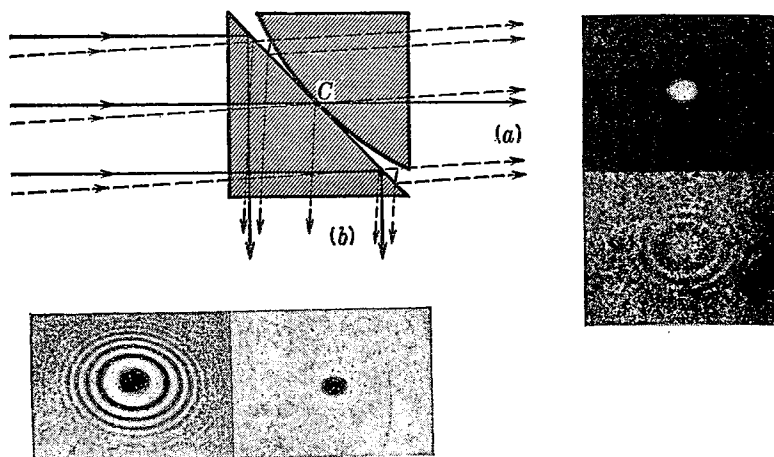


FIGURE 25K

Hall's experiment for measuring the penetration that occurs in total reflection.

25.8 METALLIC REFLECTION

In general, highly polished metallic surfaces have a higher reflectance than dielectrics. At normal incidence, for example, silver and aluminum reflect over 90 percent of all visible light. Experiments show that the reflectance depends not only on the particular metal but on the preparation of the surface and on the wavelength and direction of the incident light. If plane-polarized light is reflected from a metal at other than normal incidence (Fig. 25L), the p and s components of the incident electric vector are reflected with a phase difference and this gives rise to elliptical polarization. It is a general observation for all metals that plane-polarized light is not reflected as plane-polarized light except when it vibrates either in the plane of incidence or perpendicular to it.

In discussing the reflectance of metals it is convenient (just as for dielectrics) to resolve the incident light vector $\mathbf{E}$ into two components E_p and E_s . Curves for the two reflectances as a function of the angle of incidence are shown in Fig. 25M. These are experimental curves obtained by using white light from a tungsten-filament lamp. A comparison with the corresponding curves for a dielectric [Fig. 25B(a)] shows similarities and at the same time striking differences. Metals and dielectrics are similar

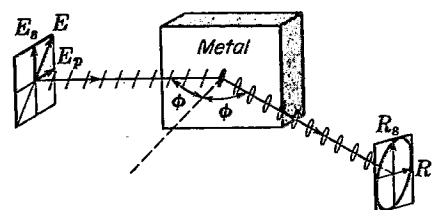


FIGURE 25L

Reflection of plane-polarized light from a metal surface to give elliptically polarized light.

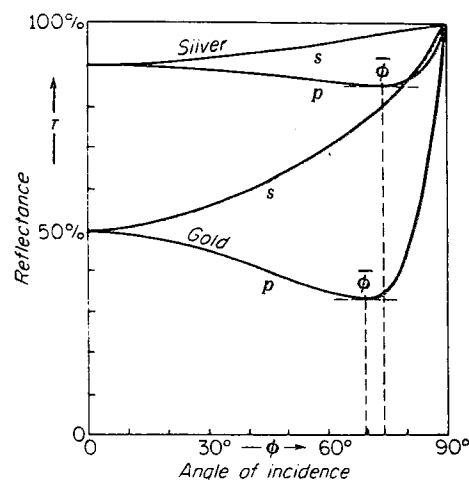
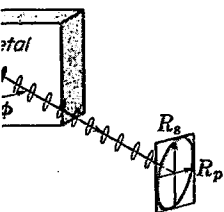


FIGURE 25M
Reflectances for plane-polarized white
light from gold and silver mirrors.

in that the values for the p and s components start together at normal incidence, separate, and then come together again at grazing incidence. The essential differences are the much higher reflectance of metals at normal incidence and the relatively high minimum at $\bar{\phi}$. This angle for minimum reflection of E_p is called the *principal angle of incidence*.

The reflectance of a metal usually varies considerably with wavelength. In Fig. 25N this variation is shown for a number of typical metals. In spite of irregularities at the shorter wavelengths, all metals reflect very well in the red and infrared. The face plates on the Apollo space suits worn by the astronauts on the moon were coated with a thin layer of gold. The coating reflected at least 70 percent of the light from the sun. Objects seen through the visor appear light-blue or green in color, but the eyes readily adapt to this color, which soon appears to be practically white. Such face plates are designed to decrease the thermal load on the suit's cooling system by strongly reflecting the infrared radiation from the sun while transmitting sufficient visible light. Gold films deposited on the surface of a sheet of plastic to be used as window shades are to be found on the sunny sides of many houses and office buildings for the same reasons.

Silver and aluminum are of particular importance for general use because they maintain their high reflectance throughout the visible spectrum. The development of methods of depositing metal films by evaporation in vacuum has rendered aluminum the most satisfactory substance for mirrors in optical instruments. This is due chiefly to two factors: (1) aluminum retains its high reflectance in the near ultraviolet as well as in the visible, and (2) the surface does not easily tarnish even after years of exposure to air. It is now standard practice to coat the mirrors of large reflecting telescopes, such as the 200-in. instrument at Mt. Palomar, with evaporated aluminum. A freshly made silver mirror actually has a slightly greater reflectance in the visible, but it soon tarnishes and becomes poorer than aluminum. For the reflecting surfaces of Fabry-Perot etalons, however, silver is preferred for use in visible and infrared



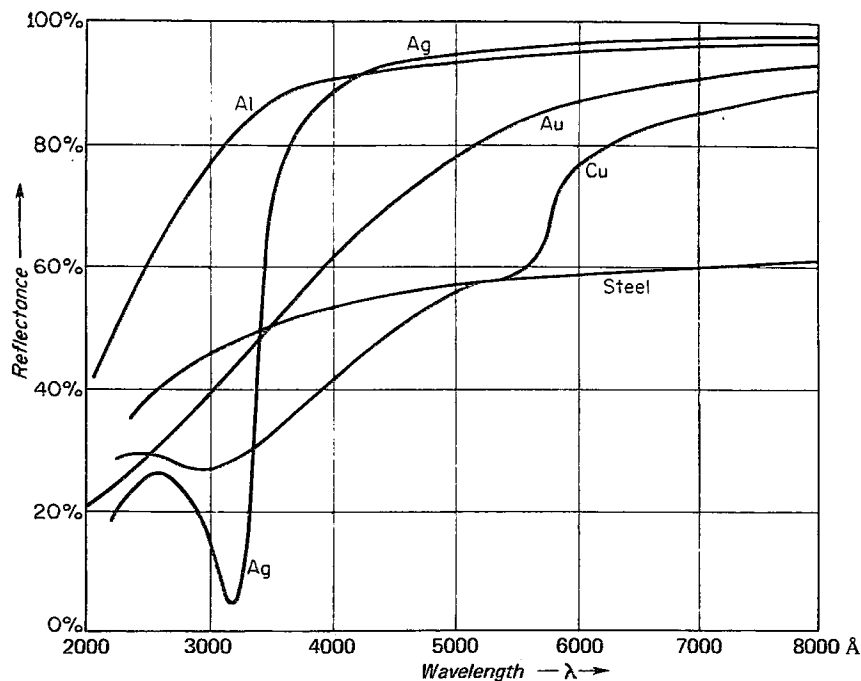


FIGURE 25N

Reflectances at normal incidence of aluminum, silver, gold, copper, and steel.

light. For ultraviolet light aluminum or a mixture of aluminum and magnesium is better.

Silver is exceptional in showing a narrow region of very low reflectance near 3200 Å. The light of this wavelength that is not reflected may be mostly transmitted if the silver film is sufficiently thin. Such a transmission band is also possessed by the alkali metals at still shorter wavelengths.* A sodium film, for example, can be used as an ultraviolet filter, opaque to all wavelengths except those near 1950 Å.

25.9 OPTICAL CONSTANTS OF METALS

The optical properties of a dielectric at a particular wavelength are completely described by one constant, the refractive index n at the wavelength. For a metal, however, another constant must be specified which measures the strength of absorption of light as it enters the metal. Because they contain free electrons, metals have very high absorption, the intensity falling to practically zero in a small fraction of a wavelength.

* For details, see R. W. Wood, "Physical Optics," 3d ed., pp. 558-566, The Macmillan Company, New York, 1934; reprinted (paperback) Dover Publications, Inc., New York, 1968.

An important quantity used in dealing with the optics of metals is the absorption index κ , which is defined in terms of the absorption coefficients κ_0 and α (Sec. 23.6) as

$$\kappa = \frac{\kappa_0}{n} = \frac{\alpha\lambda}{4\pi n} \quad (25j)$$

The determination of n for a dielectric material is usually accomplished by refraction measurements, but it can also be done using the reflected light by finding the polarizing angle and applying Brewster's law. With metals the absorption is so strong that measurements are difficult to do with transmitted light. It has been possible, working with very thin samples, to measure rough values of n and κ , but besides their inaccuracy the results may not be strictly applicable to the metal in bulk. Hence accurate values of the optical constants of metals are determined by investigating the reflected light.

Since there are the two constants to be found, n and κ , two measurements are required. In analogy to the measurement of Brewster's angle for dielectrics, one of these may be the *principal angle of incidence* $\bar{\phi}$, defined above. The other is then the corresponding azimuth, called the *principal azimuth* $\bar{\psi}$. In view of the fact that the light reflected from metals is elliptically polarized, it is not immediately evident what is meant by its azimuth. The definition is made by disregarding the phase difference between the p and s components, which is actually 90° if the light is incident at $\bar{\phi}$, and defining the azimuth in the same way as for dielectrics, namely by the equation

$$\tan \psi = \frac{R_p}{R_s} \quad (25k)$$

The theory shows that to a fair approximation* the two constants can be found from the relations

$$\begin{aligned} n \sqrt{1 + \kappa^2} &= \sin \bar{\phi} \tan \bar{\phi} \\ \kappa &= \tan 2\bar{\psi} \end{aligned} \quad (25l)$$

The method of measurement of $\bar{\phi}$ and $\bar{\psi}$ will be briefly described below, after we have considered the variation in the character of the reflected light with the angle of incidence.

The values of the optical constants found in the literature show considerable variations because of different preparation of the surfaces, purity of the samples, and accuracy of the equations used. We quote in Table 25A, however, some typical values, including in the last column the reflectances at normal incidence. It will be seen that there are large variations of n among the metals, those for the better conductors running considerably below unity. These refractive indices cannot be interpreted in the same way as for dielectrics, since here we are dealing with highly damped waves (see Sec. 23.6). The value of κ_0 for copper, for instance, corresponds to the intensity falling to $1/e$ when the light penetrates a depth of only one thirty-third of a vacuum wavelength.

* See H. Geiger and K. Schesl, "Handbuch der Physik," vol. 20, pp. 240-250, Springer-Verlag OHG, Berlin, 1928, which in general follows the work of C. Pfeiffer, "Beiträge zur Kenntnis der Metallreflexion," dissertation, Giessen, 1912.

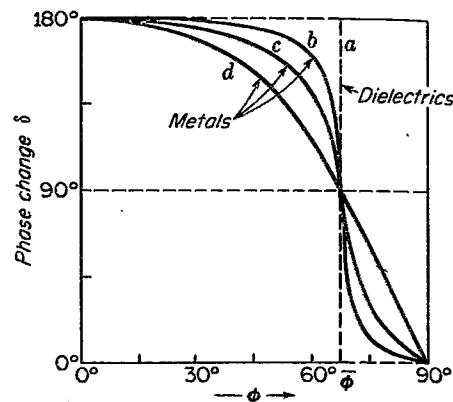


FIGURE 250

Plots of the phase difference $\delta_p - \delta_s$ for a dielectric, a , and for three metals, b , c , and d , of increasing absorption index κ .

25.10 DESCRIPTION OF THE LIGHT REFLECTED FROM METALS

If plane-polarized light is reflected from a metal, the shape and orientation of the elliptical vibration in the reflected light depend on the orientation of the incident vibration, on the magnitude of the reflected p and s components, and on the phase difference between them. The latter factor has not thus far been discussed, and a quantitative treatment of it would require a more extensive mathematical development than would be profitable to include here. We may, however, consider the main result on the behavior of δ ($= \delta_p - \delta_s$) as a function of ϕ .

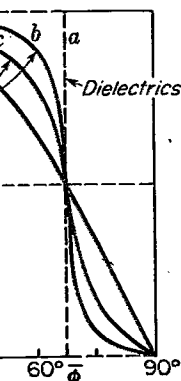
Figure 250 shows plots of the theoretical equations for the phase differences for three different metals b , c , and d , in the order of increasing absorption index κ . It also shows, as the broken line a , the plot for a dielectric, which has $\kappa = 0$. We see that the discontinuous transition of δ from π to zero which occurs at $\bar{\phi}$ for dielectrics becomes for metals a more or less gradual change. We also note that at the principal angle of incidence the value of δ is always exactly 90° .

Knowing the values of R_p/E_p , R_s/E_s , and δ , it is possible to predict the shape of the elliptical vibration reflected at each angle of incidence. Thus suppose, as in Fig.

Table 25A OPTICAL CONSTANTS FOR VARIOUS METALS FOR SODIUM LIGHT, $\lambda = 5893 \text{ \AA}$

Metal	$\bar{\phi}$	$\bar{\psi}$	n	κ	κ_0	$r, \%$
Steel*	$77^\circ 9'$	$27^\circ 45'$	2.485	1.381	3.433	58.4
Cobalt*	$78^\circ 5'$	$31^\circ 40'$	2.120	1.900	4.040	67.5
Copper*	$71^\circ 34'$	$39^\circ 5'$	0.617	4.258	2.630	74.1
Silver*	$75^\circ 35'$	$43^\circ 47'$	0.177	20.554	3.638	95.0
Gold	$72^\circ 18'$	$41^\circ 39'$	0.37	7.62	2.82	85.1
Sodium	$71^\circ 19'$	$44^\circ 58'$	0.005	522.0	2.61	99.7

* Data supplied the authors courtesy of R. S. Minor.



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κ_0	$r, \%$
3.433	58.4
4.040	67.5
2.630	74.1
3.638	95.0
2.82	85.1
2.61	99.7

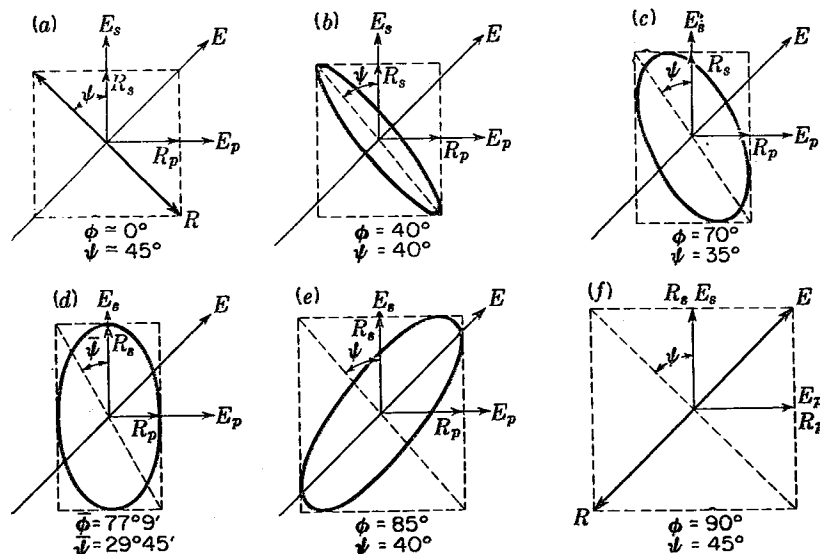


FIGURE 25P

Elliptically polarized light ($\lambda 5893$) reflected at different angles ϕ from a steel mirror.

25P, that the electric vector in the incident plane-polarized light makes an angle of 45° with the plane of incidence, so that $E_p = E_s$. We have taken the reflecting metal to be steel, which according to Fig. 25N has a reflectance $R^2/E^2 = 0.58$ for sodium light at normal incidence. Hence near normal incidence [case (a) of Fig. 25P] we construct the reflected amplitudes $R_p = R_s = 0.76E_p = 0.76E_s$, since $0.76 = \sqrt{0.58}$. Now because of the phase change of π shown in Fig. 25O, we must shift the p vibration in the reflected light 180° ahead of the s vibration, and the result is a linear vibration of amplitude R in the direction shown. This direction is actually opposite in space to that of E [see Fig. 25G(a)]. As the angle of incidence increases from zero, the gradual change in the phase difference causes the vibration to open out into an ellipse which is contained in a rectangle of sides $2R_p$ and $2R_s$. When the angle $\bar{\phi}$ is reached, as in (d) of the figure, we obtain an ellipse symmetrical to the axes, and the one that has the least eccentricity. From then on the ellipse becomes slimmer until finally at grazing incidence, as in (f), we have a linear vibration of the same amplitude as the incident light, but exactly out of phase with it.

The meaning of the azimuth angle ψ is best seen from Fig. 25P. It is the angle that the diagonal of the rectangle makes with R_s . From the figure we see that ψ first diminishes and then increases again in going from $\phi = 0$ to $\phi = 90^\circ$. The minimum value occurs at $\bar{\phi}$, but it is not zero at this angle as it is for a dielectric. The depth of this minimum becomes less for metals of larger κ . This effect may be seen in Fig. 25Q, where the letters a to d have the same significance as in Fig. 25O. In the figure we have marked the value of the principal azimuth $\bar{\psi}$ for the particular metal c .

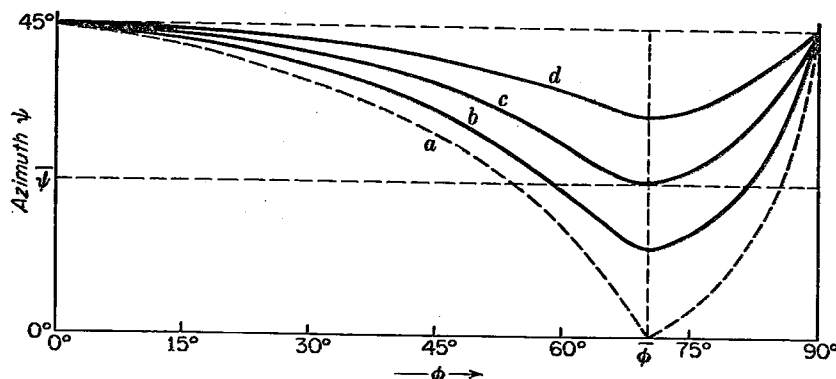


FIGURE 25Q

Azimuth angle $\bar{\psi}$ for a dielectric, a , and for three metals, b , c , and d .

25.11 MEASUREMENT OF THE PRINCIPAL ANGLE OF INCIDENCE AND PRINCIPAL AZIMUTH

The determination of these quantities is a special case of the general problem of the analysis of elliptically polarized light, a problem which will be treated in some detail in Chap. 27. It is not difficult to see, however, with the aid of Fig. 25R in connection with Fig. 25P(d), how measurements of $\bar{\phi}$ and $\bar{\psi}$ might be made. Let the nicol N_1 of Fig. 25R be oriented so that the incident vibrations are at 45° to the plane of incidence. In the reflected beam is placed some type of compensator C , which retards the p vibrations by a quarter of a period, or 90° , with respect to the s vibrations. This could be a Fresnel rhomb (Sec. 25.6) but is more commonly a quarter-wave plate or a Soleil compensator (Secs. 27.2 and 27.4). Now at any angle of incidence other than $\bar{\phi}$ the value of δ is different from 90° , so that the phase difference will not be completely removed by the compensator. The light transmitted by C will still be elliptically polarized and cannot be extinguished by rotation of the analyzer N_2 . Various angles of incidence are tried until complete extinction becomes possible, and under this condition the light is incident at $\bar{\phi}$.

The fact that it is possible to obtain complete extinction with a nicol means that the compensator has changed the elliptically polarized reflected light into plane-polarized light. In Fig. 25P(d) the ellipse is converted into a linear motion along the

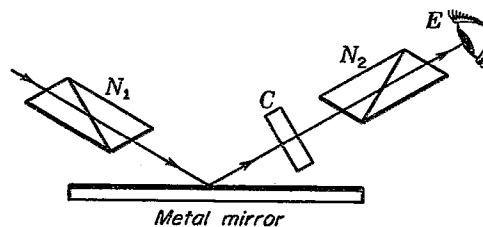
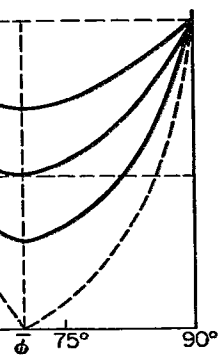


FIGURE 25R

Apparatus for determining the principle angle of incidence and principal azimuth for a metal.

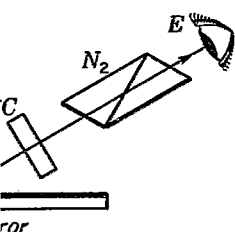


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diagonal of the rectangle by removing the phase difference of 90° that exists between the p and s components. It will therefore be seen that when the condition of extinction has been found the plane of transmission of the analyzer makes the angle $\bar{\psi}$ with R_p , namely, with the plane of incidence.

25.12 WIENER'S EXPERIMENTS

In Sec. 12.3 we described a classical experiment in which Wiener demonstrated the formation of standing waves of light by reflection from a silver mirror. The object of this experiment was not only to reveal the standing waves but also to tell whether it is the electric or the magnetic vector that produces the observed effects, and hence is to be identified as the "light vector." According to the electromagnetic theory, the incident and reflected electric vectors are oppositely directed in space for external reflection at normal incidence. With dielectrics the reflected waves have a much smaller amplitude than the incident ones so that the destructive interference is not complete. For metals, however, we should obtain a node of the electric vector at the surface.* As to the magnetic vectors, their relative directions in the incident and reflected light can be found from the fact that $\mathbf{E}$, $\mathbf{H}$, and the direction of propagation are related according to the right-hand screw rule. The result is shown in Fig. 25S. If the angle of incidence is made to approach zero, we see that the $\mathbf{H}'$ and $\mathbf{H}$ vectors approach the same direction for each polarization. Their superposition should produce an antinode of the standing waves at the surface. Now as was explained before, *Wiener observed a node where the detecting plate touched the surface. This indicated that the electric vector was the important one, at least for photographic action.*

One would expect from theory that the electric vector would be more important than the magnetic one in producing the observed effects of light. Wherever it is a question of the action of light on electrons, the electric fields in the wave exert much greater forces than the magnetic fields. In fact, only 2 years after Wiener's work Drude and Nernst showed that the same result holds when fluorescence instead of photography is used for detection. Later Ives demonstrated it using the photoelectric effect. It is assumed that the electric vector is also responsible for vision.

An even more convincing demonstration, and one which does not depend on the phase changes or on the achievement of perfect contact of the end of the photographic plate with the mirror, was given by Wiener in the following way. Plane-polarized light was reflected at an angle of incidence of exactly 45°. Then the incident and reflected rays are at right angles to each other, and the orientations of the vectors in space are those illustrated in Fig. 25S. We see that for s polarization the electric vectors E_s and R_s vibrate along the same line and can interfere. On the other hand E_p and R_p are perpendicular to each other, and no interference is possible. Exactly the reverse is true for the $\mathbf{H}$ vectors. The experiment is illustrated schematically

* The values of δ_p and δ_s are not exactly 0 or 180° for metals at normal incidence, although their difference is. The only effect of this, however, is to shift the position of the node so that it does not occur exactly at the surface. With silver, for example, the node is located 0.043λ below the surface.

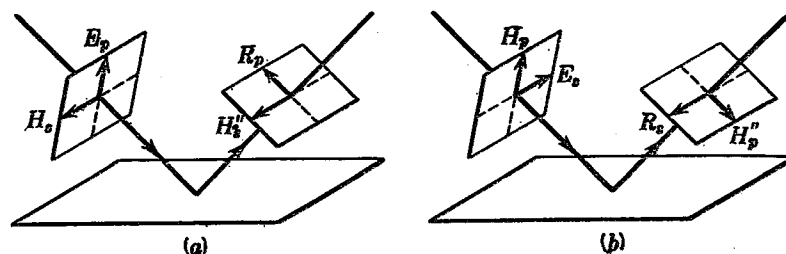


FIGURE 25S

Space relations between the incident and reflected E and H vectors: (a) for p polarization and (b) for s polarization. The angle of incidence is assumed to be less than 45° .

in Fig. 25T. In part (a) the electric vector is perpendicular to the plane of the figure, a condition which could be achieved by a preliminary reflection from a glass plate at Brewster's angle, and interference can occur along the horizontal planes marked by dots. These planes are $1/\sqrt{2}$ times as far apart as for normal incidence. In the figure the phase change of π on reflection is indicated by the change from solid to broken lines, and vice versa. For the corresponding magnetic vectors, as shown in part (b), there is no phase change on reflection. On the surface at point A the resultant is a linear vibration normal to the surface. Farther up the vibration becomes elliptical, then circular as at a , and finally linear again at B with horizontal vibrations. The reverse sequence is followed to point C , with the points A , B , and C being separated by a distance $\lambda/2$ along the ray. The energy associated with all these vibration forms is the same (Sec. 28.8), and hence, if the magnetic vector were the active one, the test plate should be uniformly blackened. Wiener actually found interference bands in the case illustrated, and uniform blackening when the vibrations in the incident light were turned through 90° .

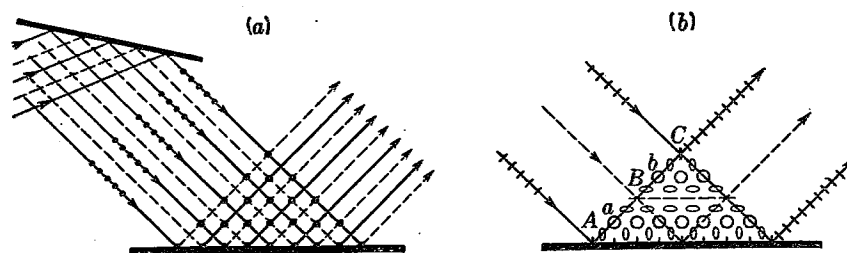
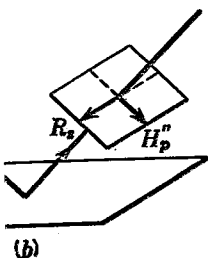


FIGURE 25T

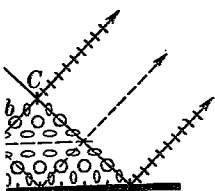
Wiener's experiment at 45° incidence. Interference is observed for the electric vector oriented as in (a), while the corresponding magnetic vector (b) would show none.



(b) H vectors: (a) for p plane is assumed to be

plane of the figure, on a glass plate at planes marked by incidence. In the figure, a solid to broken line shown in part (b), the resultant is a becomes elliptical, al vibrations. The C being separated use vibration forms active one, the test erence bands in the incident light were

(b)



reserved for the electric vector (b) would

PROBLEMS

- 25.1 Plot the external reflection intensity curves for red light incident on a transparent crystal of diamond. Use the refractive index given in Table 23A.
- 25.2 Plot the internal reflectance curves for red light in diamond. Use the refractive index given in Table 23A.
- 25.3 Compute the reflectance at normal incidence for the following materials: (a) diamond, $n = 2.426$; (b) quartz, $n = 1.547$; (c) rutile, $n = 2.946$; (d) crown glass, $n = 1.526$; (e) metallic silver, $n = 0.177$, $\kappa_0 = 3.638$; (f) steel, $n = 2.485$, $\kappa_0 = 3.433$.
Ans. (a) 17.32%, (b) 4.61%, (c) 24.32%, (d) 4.34%, (e) 95.16%, (f) 58.46%
- 25.4 (a) Derive an equation for the azimuth of the refracted light in a dielectric. Assuming $n = 1.50$, (b) make a plot of this angle ψ' against ϕ' , similar to that for reflected light in Fig. 25H.
- 25.5 Plane-polarized light is incident at $\phi = 70^\circ$ on a glass surface, with its electric vector vibrating at 30° with the plane of incidence. Assuming $n = 1.750$, calculate (a) the polarizing angle, (b) the critical angle, (c) the relative magnitudes E_p to E_s , (d) the relative magnitudes of R_p and R_s , and (e) the azimuth angle ψ .
- 25.6 Plane-polarized light is internally reflected at $\phi = 45^\circ$ from the hypotenuse of a total-reflection prism made of glass of index 1.650. If the azimuth of the incident light is 45° , calculate (a) the phase change of the p and s components and (b) the phase difference between the p and s components. (c) Plot to scale the elliptical vibrations form as was done in Fig. 25J.
- 25.7 Unpolarized light strikes a smooth glass surface at an angle of 35° . Assume the glass index to be 1.750. Calculate (a) the amplitudes and (b) the intensities of the reflected p and s components. (c) Find the degree of polarization of the refracted light (see Sec. 24.4).
Ans. (a) amplitudes, 0.2055 and 0.3374, (b) reflectances, 0.04223 and 0.11384, (c) polarization 3.884%
- 25.8 (a) Make a plot of the phase changes on internal reflection in glass having an index of 1.825. Limit the plot to angles between the critical angle and grazing incidence. (b) Take the difference $\delta = \delta_p - \delta_s$, and find what two angles could be used in designing a Fresnel rhomb of this glass.
- 25.9 Explain why in making a Fresnel rhomb described in Sec. 25.6, it is more desirable to choose the angle $54^\circ 37'$ than the other angle ($48^\circ 37'$), which also gives a phase difference of $\delta = 45^\circ$.
- 25.10 The optical constants for a given polished metal surface are $n = 2.340$ and $\kappa = 1.176$, for green light. Calculate (a) its reflectance at normal incidence, (b) its principal angle of incidence, and (c) its principal azimuth. *Ans.* (a) 50.0%, (b) 75.0° , (c) 24.81°
- 25.11 For a certain polished metal surface the principal angle of incidence is measured to be 65.5° and the principal azimuth 38.4° . Determine (a) the optical constants of this metal and (b) its reflectance at normal incidence.