Analysis of the vacuum cannon

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We develop a model for the velocity of a projectile in a vacuum cannon. The theoretical maximum velocity is independent of the vacuum cannon diameter and projectile mass and is significantly lower than the speed of sound. Experimental measurements support the theory as an upper limit.
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I. INTRODUCTION

The vacuum cannon, or vacuum bazooka as it is sometimes called, is a spectacular classroom demonstration of the very real nature of air pressure.[1] A 1–2 m length of PVC pipe, with a T fitting to facilitate attachment to a vacuum pump, is loaded with one or more ping-pong balls (see Fig. 1). The pipe is then sealed with caps of aluminum foil or plastic tape.[2] The apparatus is evacuated, and the foil cap nearest the ping-pong balls is popped. The ping-pong balls are driven out of the other end of the pipe by atmospheric pressure at a startlingly high velocity.

In the course of discussion about this apparatus with fellow physics professors, it became evident that there was no clear understanding of the maximum theoretical velocity such an apparatus should produce. The zeroth-order approximation is that the acceleration of the projectile would be a constant,

$$a_0 = \frac{F}{m} = \frac{P_0 A}{m}, \tag{1}$$

where P_0 and A are the atmospheric pressure and projectile cross-sectional area, respectively. Equation (1) predicts a maximum speed which is dependent on the tube length[3] L:

$$v(L) = \sqrt{\frac{2P_0 AL}{m}}. \tag{2}$$

This estimate runs into problems for longer barrels for which it predicts a speed greater than the speed of sound. The ensuing ‘‘common-sense correction’’ is that the projectile asymptotically approaches the speed of sound.

II. ANALYSIS

The actual situation is more complicated, because the air pressure must accelerate not only the projectile, but also the air column behind the projectile. A closed form solution exists, however. Consider a projectile of mass m and cross section A in a horizontal vacuum cannon. The projectile begins at position $x=0$. We will assume for this first-order calculation that the projectile fits and seals the barrel perfectly, and that effects due to friction, viscosity, and compressibility are negligible. We also assume that the pressure at $x=0$ remains constant at P_0.

From Newton's second law, we obtain

$$P_0 A = \frac{d}{dt}[(m+\rho x A)v], \tag{3}$$

which can be integrated directly to obtain

$$P_0 A t = m\left(1+\frac{\rho x A}{m}\right)v, \tag{4}$$

where ρ is the air density and x is the position of the projectile. (The integration constant is zero if the initial velocity is taken to be zero.)

If we define for convenience a characteristic length[4] $\lambda \equiv m/\rho A$, Eq. (4) may be rewritten as

$$P_0 A t = m\left(1+\frac{x}{\lambda}\right)v. \tag{5}$$

The integral of Eq. (5) gives

$$\frac{1}{2}P_0 A t^2 = m\left(x+\frac{x^2}{2\lambda}\right). \tag{6}$$

The solution of Eq. (6) for x is

$$x(t) = \lambda\left[\sqrt{1+\frac{a_0 t^2}{\lambda}}-1\right], \tag{7}$$

where we have substituted a_0 for the collection of constants. The velocity is thus

$$v(t) = \frac{dx}{dt} = \frac{a_0 t}{\sqrt{1+\frac{a_0 t^2}{\lambda}}}. \tag{8}$$

The initial acceleration of the projectile is

$$\left.\frac{dv}{dt}\right|_{t\to 0} = a_0 = \frac{P_0 A}{m}, \tag{9}$$

which is the result one would expect from the zeroth-order approximation, Eq. (1). As an aside, it is interesting to note that the magnitude of this acceleration for a ping-pong ball is approximately 4700 g's!

By factoring $\sqrt{a_0 t^2/\lambda}$ out of the denominator of Eq. (8), we may express the velocity in the asymptotic form:

$$v(t) = v_{\max}\left(1+\frac{\lambda}{a_0 t^2}\right)^{-1/2}, \tag{10}$$

where

$$v_{\max} \equiv \sqrt{a_0\lambda} = \sqrt{\frac{P_0}{\rho}}. \tag{11}$$

For comparison, the speed of sound is[5] $v_s = \sqrt{(\gamma P_0/\rho)}$, with $\gamma = C_p/C_v$.

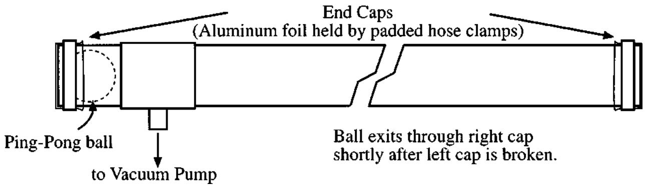

Fig. 1. Vacuum cannon.

If we substitute Eq. (6) into Eq. (10), we can determine the velocity as a function of displacement:

$$v(x)=v_{\max}\left[\frac{x}{x+\lambda}\left(1+2\,\frac{\lambda}{x}\right)\right]^{1/2}. \tag{12}$$

Equation (12) is plotted as the first-order model in Fig. 2, using the mass and cross-sectional area of a ping-pong ball (2.5 g, and 1.13×10^{-3} m^2, respectively). For comparison, the ''zeroth-order'' model [Eq. (2)], and various other parameters also are shown.

The energy of the projectile depends on the projectile mass and on the length of the cannon. For a given length L,

$$E=\frac{1}{2}mv(L)^2=\frac{1}{2}mv_{\max}^2\left(\frac{L}{L+\lambda}\right)^2\left(1+2\,\frac{\lambda}{L}\right). \tag{13}$$

We substitute for λ and $v_{\max}$ and obtain

$$E=\frac{1}{2}m\,\frac{P_0}{\rho}\left[1-\left(1+\frac{\rho AL}{m}\right)^{-2}\right]. \tag{14}$$

If we take the limit of this energy for large mass, we obtain, to nobody's surprise,

$$\lim_{m\to\infty} E=P_0AL, \tag{15}$$

which is the energy required to evacuate the vacuum cannon in the first place. The momentum of the projectile, $p=mv(L)$, is not limited, but increases (in the limit of large masses) as the square root of the mass.

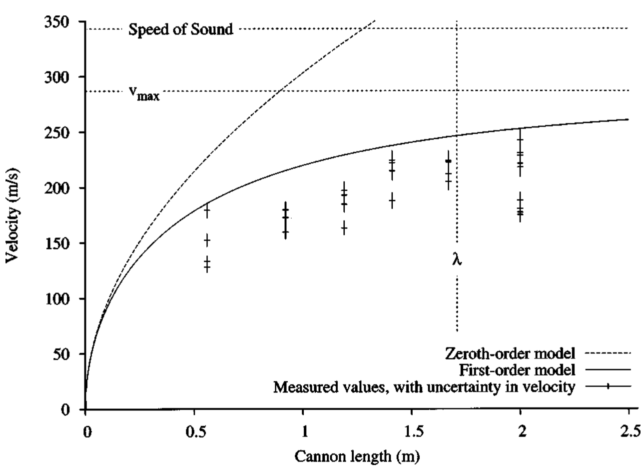

Fig. 2. Comparison of experimental measurements with various models for vacuum cannon velocity.

III. EXPERIMENTAL VALIDATION

The experimental values shown in Fig. 2 were obtained by firing the ping-pong balls through two PASCO[6] photogates at a fixed spacing. The photogate signals were sent to an HP 5600B digital oscilloscope and to an HP 5300A timer. The oscilloscope was used to determine whether the shot was ''good,'' that is, whether there were two distinct photogate events at about the right time interval, and the timer was used to obtain a more precise value for the time of flight between the gates.

There is considerable scatter in the data, due to unavoidable random factors in the firing and measurement process. The end-cap does not collapse instantaneously, and sometimes a significant area of foil around the edge may remain and interfere with the incoming airflow. This interference lowers the ball velocity by an unpredictable amount. The ball comes out with wildly varying amounts of spin, judging by the unpredictable curve of its subsequent trajectory. If this spin arises from friction with the tube wall during transit, the amount of energy lost to friction also must vary considerably from shot to shot, and this loss contributes to the scatter in the measured data. Finally, there is the inherent uncertainty in the measurement of the position of a spherical object by a photogate. Depending on the exact path of the ball between the two gates, the effective gate position could vary by more than a centimeter at each gate.

There are several weaknesses to the model described in Eqs. (10) and (12). It does not take into account any ''blowby'' of air past the ball. The ping-pong ball is not a perfect fit in the PVC pipe: there is a 1.2-mm gap around the edge. Any such blowby would result in a buildup of air pressure in front of the ball which would slow the ball, although this effect has not been experimentally quantified. The approximation does not take into account any drop in the air pressure at $x=0$ during the firing process. Such a drop would lower the final velocity of the ball. Finally, it assumes that ρ remains at its constant equilibrium value in the tube during the firing process. It is not immediately apparent what effect the non-equilibrium value of ρ in the tube would have, but all the other sources of error—including those deliberately not included in the model, such as the impact of the ball with the exit end-cap—would lower the measured ball velocity. The experimental data shown in Fig. 2 are consistent with the model giving an upper limit to the velocity.

IV. A CAUTIONARY NOTE

A ping-pong ball slows down very rapidly in air, so if you were to get hit with one of these from across a room it would not be a problem. However, at the muzzle of the vacuum cannon the ping-pong ball could be moving as fast as $\sqrt{(P_0/\rho)}=287$ m/s. The kinetic energy of a ping-pong ball at this velocity is higher than that of a bullet from most .22 caliber handguns,[7] so be careful where you point this thing. When doing this demonstration it is better to load the cannon with three to six balls instead of one. The demonstration is still just as impressive as a single ball, and the increased mass lowers the exit velocity (for a 1–2-m cannon) to a much safer level.

V. CONCLUSIONS

This model predicts that the maximum velocity available to a vacuum cannon is not the speed of sound v_s, but

$v_s/\sqrt{\gamma}$. This asymptotic limit is independent of the vacuum cannon length, diameter, and projectile mass. The measured projectile velocity is well below this limit. The kinetic energy of the projectile is limited, for a given cannon length, although the momentum is not.

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[1]Physics Instructional Resource Association (PIRA) Demonstration Classification Scheme 2B30.70.

[2]J. Cockman, ''Improved Vacuum Bazooka,'' Phys. Teach. **41**, 246–247 (2003).

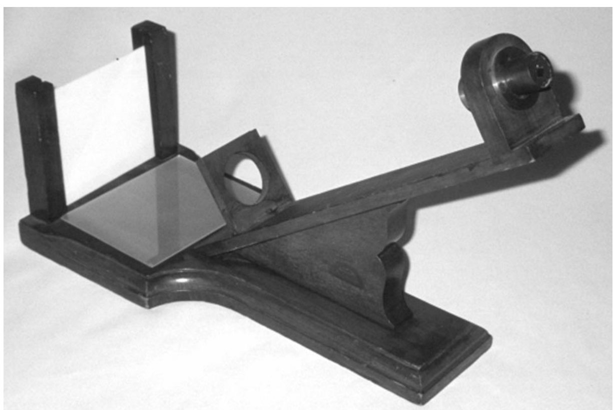
[3]N. A. Downie, *Vacuum Bazookas, Electric Rainbow Jelly, and 27 Other Saturday Science Projects* (Princeton U. P., New York, 2001).

[4]This definition was chosen purely to simplify the analysis. Interestingly enough, it also is the length of an air column with a mass equal to the mass of the projectile.

[5]R. Feynman, R. B. Leighton, and M. L. Sands, *The Feynman Lectures on Physics* (Addison–Wesley, Reading, MA, 1989), Vol. 1.

[6]PASCO Scientific, ⟨http://www.pasco.com⟩, part #ME-9498A.

[7]The ballistics of many guns can be found at ⟨http://www.volny.cz/buchtik/Revo/Ballistic_Info_komplet.htm⟩.



Pickering Polariscope. This form of polariscope was described in 1873 by Edward C. Pickering, at that time at M.I.T. Light illuminates the ground glass screen on the left, and is reflected from the horizontal glass plate at Brewster's angle. The resulting plane-polarized light passes through the sample held in the holder on the angled bracket. The eyepiece contains a Nicol prism that acts as a second polarizer. In use, the transmission axes of the two polarizers are set at right angles to each other, and birefringent samples made of thin slices of mica mounted on glass are placed in the holder. This example was made by the Ziegler Electric Company of Boston, and is in the Greenslade Collection. (Photograph and notes by Thomas B. Greenslade, Jr., Kenyon College)